LINEAR CLOSED-LOOP SYSTEMS

THE CONTROL SYSTEM

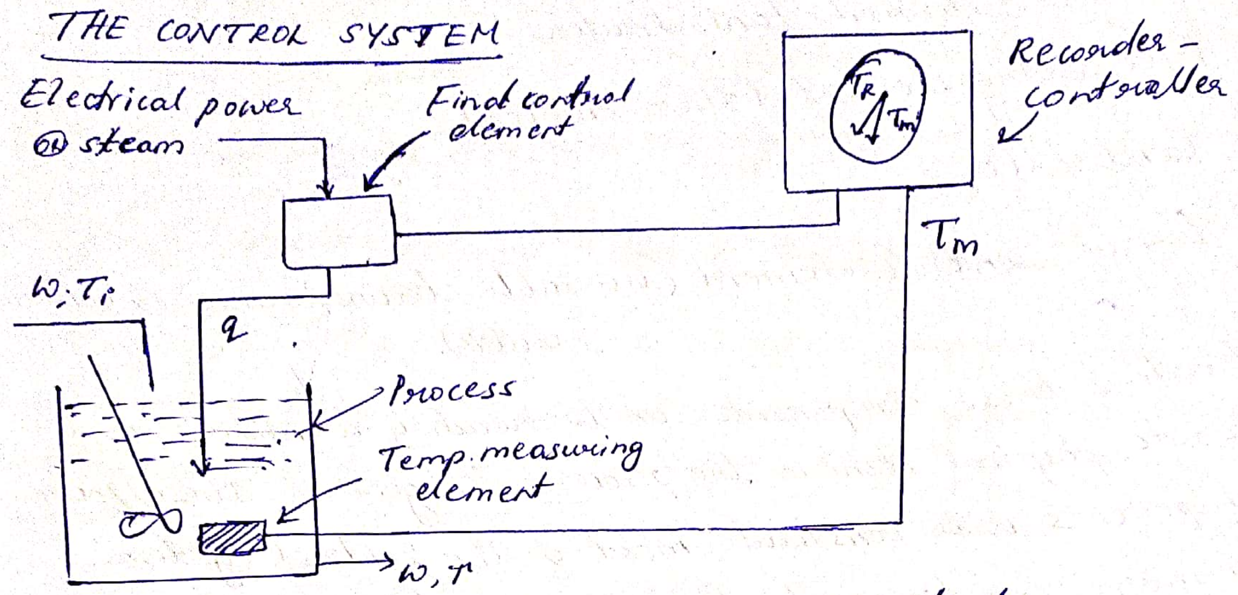


Fig- Control system for a stirred-tank heater

Consider a stirred-tank heater as a system as shown in fig. A liquid stream at a temperature T_i enters an insulated, well-stirred tank at a constant flow rate w (mass/time). Temp. in the tank should be controlled OR maintained at desired T_R by means of controller.

If the measured tank temp. T_m differs from the desired temp. T_R, the controller senses the difference OR error, $\epsilon = T_R - T_m$, and changes the heat input to the tank by an amount that is proportional to ϵ called proportional control. The source of heat input q may be electricity OR steam. If an electrical source were used, the final control element might be a variable transformer that is used to adjust current to a resistance heating element; if steam were used, the final control element would be a control valve that adjusts the flow of steam. In either case, the output signal from the controller should adjust q in such a way as to maintain control of the temp. in the tank.

Components of a control system.

1. Process (stirred-tank heater)
2. Measuring element (thermometer)
3. Controller
4. Final control element (variable transformer @ control valve).

Each of these components can be readily identified as a separate physical item in the process. In general, these four components will constitute most of the control systems. For more complex control systems exist more components are used.

Ex- There are some processes which require a cascade control system in which two controllers and two measuring elements are used.

Block Diagram

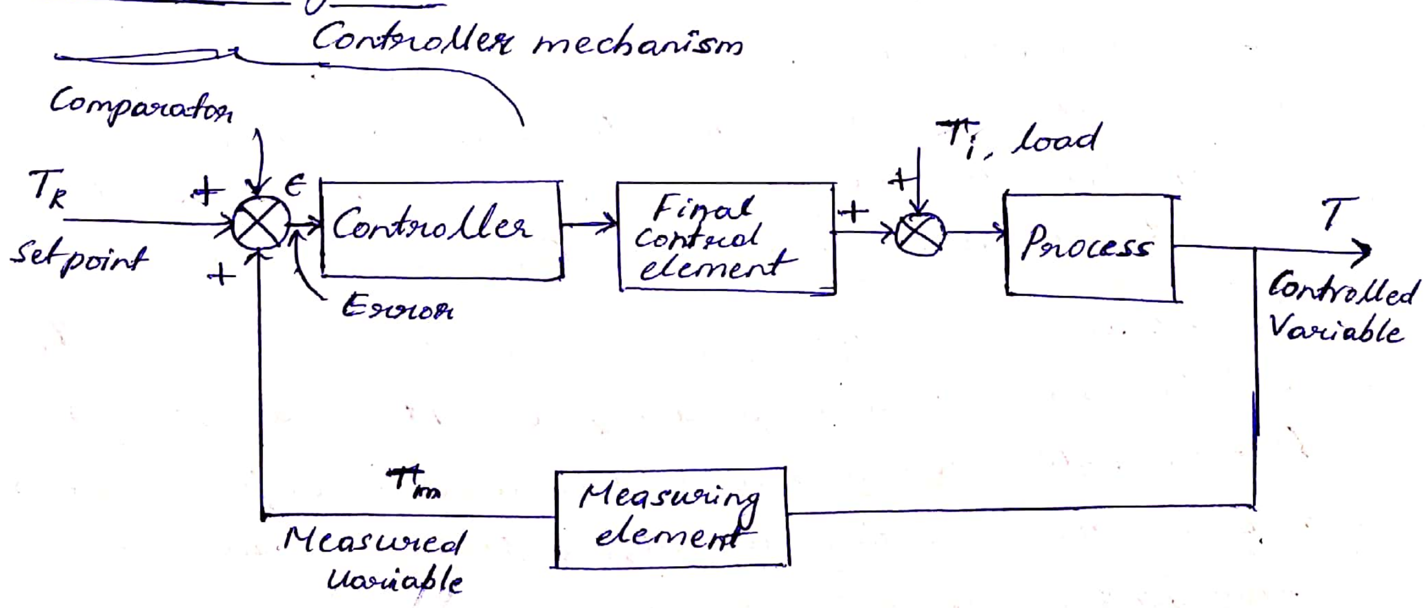


Fig - Block diagram of a simple control system

For computational purposes, it is convenient to represent the control system by means of the block diagram as shown in fig. Such a diagram makes it much easier to visualize the relationships among the various signals.

Setpoint - is the desired value of the controlled variable.

Load - The load refers to a change in any variable that may cause the controlled variable of the process to change.

Ex - Inlet temp. 'T_i' is a load variable.

Changes in flow rate

Heat loss from the tank

The control system shown in above fig is called a closed-loop system (or) a feedback system because the measured value of the controlled variable is returned (or) "fed back" to a device called the "comparator."

In the comparator, the controlled variable is compared with the desired value (or) set point. If there is any difference between the measured variable and the set point, an error is generated. This error enters a controller, which in turn adjusts the final control element in order to return the controlled variable to the set point.

Negative Feedback:

The feed back principle, which is illustrated by fig. (block diagram of a simple control system) involves the use of the controlled variable T to maintain itself at a desired value T_R. The arrangement of the apparatus is often described as negative feedback.

Negative feedback ensures that the difference between T_R and T_m is used to adjust the control element so that the tendency is to reduce the error.

Ex - Assume that the system is at s.s. and that
$T = T_m = T_R$.
If load T_i should increase, T and T_m would start to increase, which would cause the error ϵ to become negative. i.e $\epsilon = T_R - T_m = -ve$

With proportional control, the decrease in error ϵ would cause the controller and final control element to decrease the flow of heat to the system with the result that the flow of heat would eventually be reduced to a value such that T approaches T_R.

Positive Feedback :

If the signal to the comparator were obtained by adding T_R and T_m, ~~the~~ It is positive feedback which is inherently unstable.

Assume that the system is at steady state and that $T = T_m = T_R$. If T_i were to increase, T and T_m would increase, which would cause the signal the comparator ϵ to increase, with the result that the heat to the system would increase.

However, this action, which is just the opposite of that needed, would cause T to increase further. It should be clear that this situation would cause T to "run away" and control would not be achieved.

In complex systems positive feed back arises naturally. But in most systems positive feed back would never be used intentionally in the system.

Servo Problem versus Regulator Problem

In servoproblem, it is assumed that there is no change in load T_i and that is required change in bath temp. according to some prescribed function of time.

For this problem i.e. block diagram – the set point T_R is changed in accordance with the desired variation in bath temp. If the variation in T_R sufficiently slow, the bath temp may be expected to follow the variation in T_R very closely. There are occasions when a control system in the chemical industry will be operated in the same manner.

Ex – Varying the temp. of a reactor according to a prescribed time - temp. pattern.

- The tracking of missiles and aircraft
- Automatic machining of intricate parts from a master pattern.

Regulator Problem – In this case, the desired value T_R is to remain fixed and the purpose of the control system is to maintain the controlled variable at T_R in spite of change in load T_i.

Ex – Regulator problems are common in chemical industry and complicated industrial process.

Complicated industrial process will often have many self contained control systems, each of which maintains a particular process variable at a desired value.

Note – Change in set point – servo problem (load constant)

Change in load – Regulat problem (setpoint constant)

Development of Block Diagram:

Each block represents the functional relationship existing b/w the input and output of a particular component.

Initially, input and output relations were developed in the form of transfer functions. In block-diagram representations of control systems, the variables selected are deviation variables, and inside each block is placed the transfer function relating the input-output pair of variables. Finally, the blocks are combined to give the overall block diagram.

Process:

Consider the first block for the process. This block the input variables will differ but t.f. remains same.

To get T.F an unsteady-state energy balance around the tank gives,

$$q + wC(T_i - T_0) - wC(T - T_0) = \rho C V \frac{dT}{dt} \quad \text{——} ①$$

where, T_0 - is the difference temp.

At s.s, dT/dt - is zero, eqn ① is

$$\therefore \; q_s + wC(T_{is} - T_0) - wC(T_s - T_0) = 0 \quad \text{——} ②$$

Subtracting eq. ② from ①

$$q - q_s + wC[(T_i - T_{is}) - (T - T_s)] = \rho C V \frac{d(T - T_s)}{dt} \quad \text{——} ③$$

Introducing the deviation variables.

$$T_i' = T_i - T_{is}$$

$$Q = q - q_s$$

$$T' = T - T_s$$

eq ③ becomes,

$$Q + wC(T_i' - T') = \rho C V \frac{dT'}{dt} \quad \text{——} ④$$

Taking the Laplace transform of eq. 4 gives

$$Q(s) + wC\left[T_i'(s) - T'(s)\right] = \rho C V s T'(s) \quad \text{——— (5)}$$

or

$$T'(s)\left(\frac{\rho V}{w}s + 1\right) = \frac{Q(s)}{wC} + T_i'(s) \quad \text{——— (6)}$$

$$T'(s) = \frac{1/wC}{\tau s + 1} Q(s) + \frac{1}{\tau s + 1} T_i'(s) \quad \text{——— (7)}$$

where,

$$\tau = \frac{\rho V}{w}$$

If there is a change in $Q(t)$ only, then $T_i'(t) = 0$ and the transfer function relating T' to Q is

$$\frac{T'(s)}{Q(s)} = \frac{1/wC}{\tau s + 1} \quad \text{——— (8)}$$

If there is a change in $T_i'(t)$ only, then $Q(t) = 0$ and the transfer function relating T' to T_i' is

$$\frac{T'(s)}{T_i'(s)} = \frac{1}{\tau s + 1} \quad \text{——— (9)}$$

Eqn 7 is represented by the block diagram as shown in fig. and is simply an alternate way to express eq 7 in terms of the t. f. of eqs. 8 and 9 is also shown by this diagram.

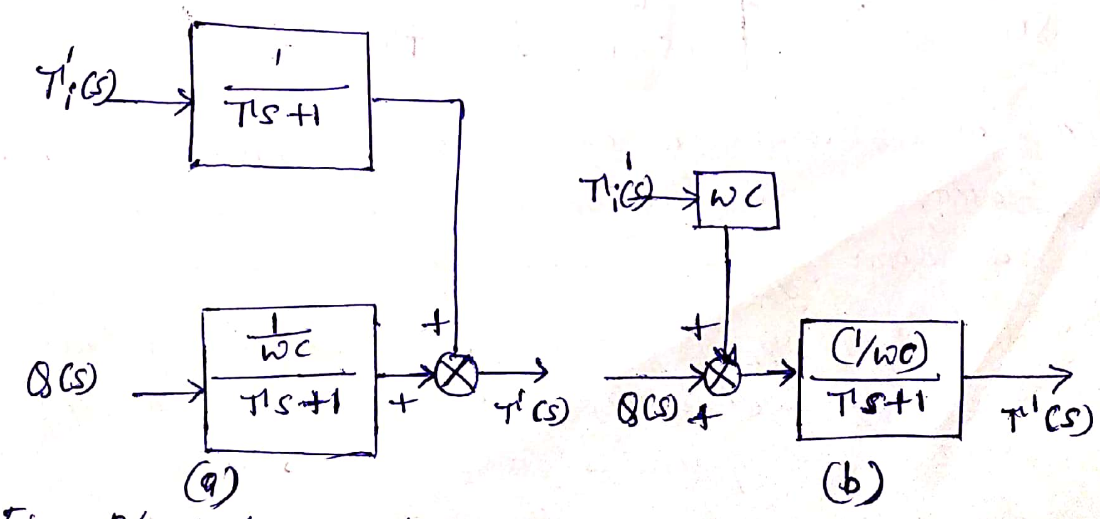


Fig - Block diagram for process

Superposition makes this representation possible.
Block diagram for process in fig. is shown indicated summation of signals by shown in fig. is called summing junction.

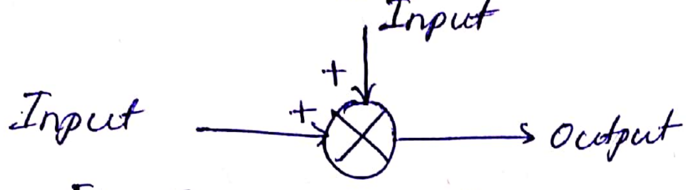


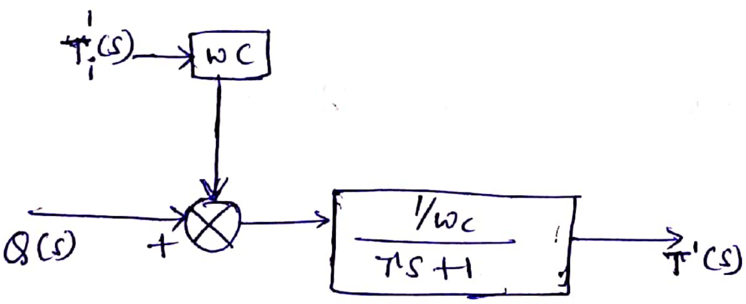
Fig- Summing junction

Subtraction is indicated with this symbol by placing a minus sign at the appropriate input. This symbol, which is standard in the control literature, may have several inputs but only one output.

A block diagram that is equivalent to previous ~~block~~ fig is shown below. This diagram is correct by rearranging as eq, thus

$$T'(s) = \left[Q(s) + wCT_i'(s)\right]\frac{1/wC}{\tau s+1} \quad \text{———} \quad (10)$$

In fig, the input variables $Q(s)$ and $wCT_i'(s)$ are summed before being operated on by the transfer function $\frac{1/wC}{\tau s+1}$.



The physical situation that exists for the control system if steam heating is used requires more careful analysis to show that above fig. is an equivalent block diagram.

Assume that a supply of steam at constant conditions is available for heating the tank. One method for introducing heat to the system is to let the steam flow through a control valve and discharge directly into the water in the tank, where it will condense completely and become part of the stream leaving the tank.

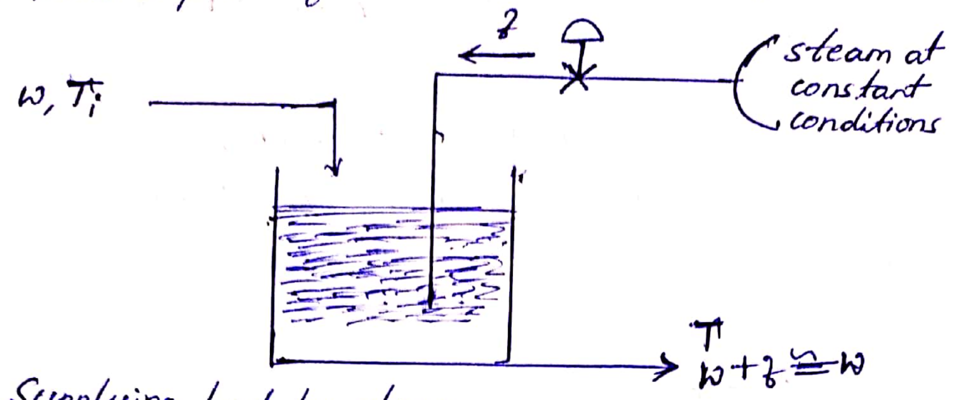


Fig - Supplying heat by steam

If the flow of steam, q is small compared with the inlet flow w, the total outlet flow is approximately equal to w. When the system is at steady state, the heat balance is written as,

$$wC(T_{is} - T_0) - wC(T_s - T_0) + q_s(H_g - H_{ls}) = 0 \quad \text{——} \quad (11)$$

where,

T_0 - reference temp. used to evaluate enthalpy of all streams entering and leaving tank.

H_g - specific enthalpy of the steam supplied, a constant.

H_{ls} - specific enthalpy of the condensed steam flowing out at T_s, as part of the total stream.

The term H_{ls} written in terms of heat capacity and temp. thus

$$H_{ls} = C(T_s - T_0) \quad \text{——} \quad (12)$$

For ~~an~~ s.s temp. changes H_{ls} changes.

$$q_s(H_g - H_{ls}) = q_s \quad \text{——} \quad (13)$$

For unsteady-state operation, q is much less than w and the temp. T of the bath does not deviate significantly from the steady state temp. T_s.

For these conditions, the unsteady-state balance is,

$$wC(T_i - T_0) - wC(T - T_0) + f(H_g - H_{ls}) = \rho C V \frac{dT}{dt} \quad \text{—— (14)}$$

If the temp. of the bath, T never ~~devo~~ deviates from T_s by more than 10°, the error in using the term $f(H_g - H_{ls})$ instead of $f(H_g - H_l)$ will be no more than 1%.

$$\therefore \; q = f(H_g - H_{ls}) \quad \text{—— (15)}$$

Therefore, q is proportional to the flow of steam f, which may be varied by means of a control valve. Both f and the deviation in T must be small. The smaller they become, the more closely eq. (14) represents the actual physical system. An exact analysis of the problem leads to a differential eq. with time-varying coefficients, and the transfer-function approach does not apply. The problems becomes considerably more difficult. Linearization techniques will provide approximation results.

Measuring Element:

The temp.-measuring element, which senses the bath temp T and transmits a signal T_m to the controller, may exhibit some dynamic lag. From mercury thermometer, this lag is ~~first order is~~ observed is first order.

Assume that the temp-measuring element is a first-order system, for which the transfer function is,

$$\frac{T_m'(s)}{T'(s)} = \frac{1}{\tau_m s + 1} \quad \text{——}$$

where, the input-output variables T' and T_m' are deviation variables, defined as,

$$T' = T - T_s$$

$$T_m' = T_m - T_{ms}$$

At s.s, $T_s = T_{ms}$ i.e. temp-measuring element reads the true bath temp. The measuring element is represented by the block diagram,

$$T'(s) \longrightarrow \boxed{\frac{1}{\tau_m s+1}} \longrightarrow T_m'(s)$$

Fig - Block diagram of measuring element.

Controller and Final Control Element

Usually, the blocks representing the controller and the final control element are combined into one block.

The overall response between the error and the heat input to the tank is considered, and assumed that the controller is a proportional controller.

The relationship for a proportional controller is,

$$\therefore \quad q = K_c \epsilon + A \qquad \text{——— (1)}$$

where, $\epsilon = T_R - T_m$

T_R - set-point temp

K_c - proportional sensitivity (or) controller gain

A - heat input when $\epsilon = 0$

At s.s, it is assumed that set point, the process temp. and measured temp. are all equal to each other, thus

$$T_{Rs} = T_s = T_{ms} \qquad \text{——— (2)}$$

T_{Rs} - set point temp. at s.s

T_s - process temp. at s.s

T_{ms} - measured temp. at s.s.

Let ϵ' be the deviation variable for error, thus.

$$\epsilon' = \epsilon - \epsilon_s \quad \text{——— (3)}$$

where,

$$\epsilon_s = T_{Rs} - T_{ms}$$

since $T_{Rs} = T_{ms}$, $\epsilon_s = 0$ and eq. (3) becomes

$$\epsilon' = \epsilon - 0 = \epsilon \quad \text{——— (4)}$$

This result shows that ϵ is itself a deviation variable.

Since $\epsilon_s = 0$, eq (1) at steady state

$$q_s = K_c \epsilon_s + A = 0 + A = A$$

Eq (1) in terms of q_s, thus

$$q = K_c \epsilon + q_s$$

OR

$$Q = K_c \epsilon \quad \text{——— (5)}$$

where, $Q = q - q_s$

The transform of eq (5) is simply

$$Q(s) = K_c \epsilon(s) \quad \text{——— (6)}$$

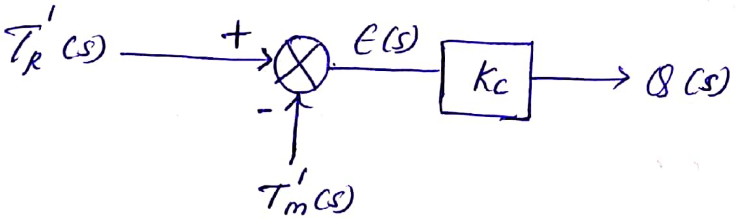


Fig- Block diagram of proportional controller.

Note that ϵ, which is also equal to ϵ', is expressed as,

$$\epsilon = T_R - T_{RS} - (T_m - T_{ms}) \quad \text{——— (7)}$$

OR

$$\epsilon = T_R' - T_m' \quad \text{——— (8)}$$

Eq (7) follows from the definition of ϵ and the fact that $T_{RS} = T_{ms}$. Taking the transform of eq (8) gives,

$$\epsilon(s) = T_R'(s) - T_m'(s) \quad \text{——— (9)}$$

The transfer function for the proportional controller given by eq. (6) and the generation of error given by eq (9) may be expressed by the block diagram.

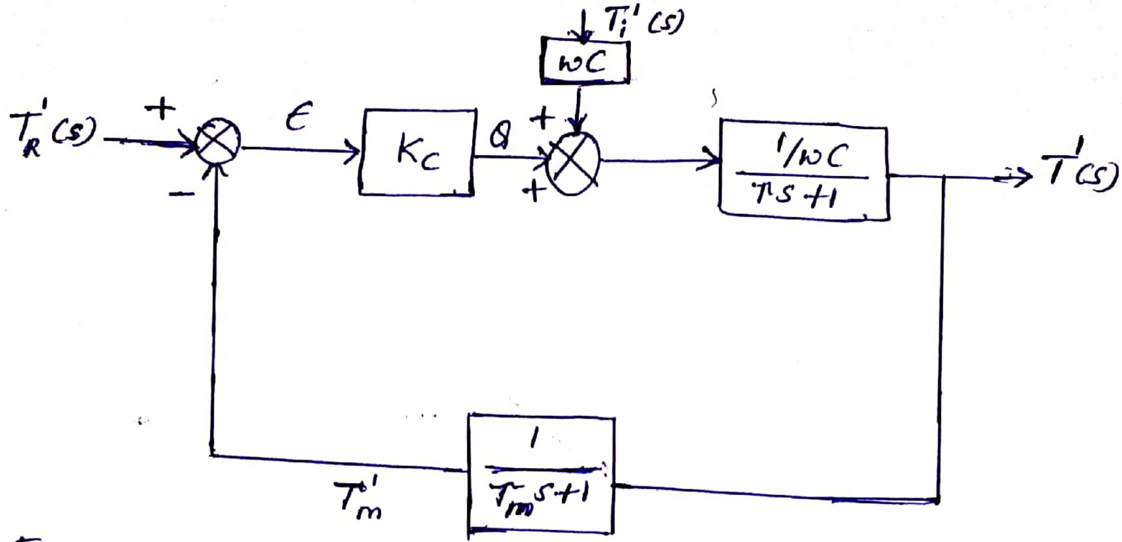


Fig- Block diagram of control system

Summary:

A control system can be translated into a block diagram that includes the transfer functions of the various components. It should be emphasized that a block diagram is simply a systematic way of writing the simultaneous differential and algebraic equations that describe the dynamic behavior of the components. The block diagram clarifies the relationships among the variables of these simultaneous equations. Another advantages of the block-diagram representation is that it clearly shows the feedback relationship between measured variable and desired variable and how the difference in these two signals (the error ϵ) is used to maintain control. A set of equations generally does not clearly indicate the relationships shown by the block diagram.

CONTROLLERS AND FINAL CONTROL ELEMENTS

The input signal to the controller is the error and the output signal of the controller is fed to the final control element. In many process control systems, this output signal is an air pressure and the final control element is a pneumatic valve that opens and closes as air pressure on the diaphragm changes.

For the mathematical analysis of control systems, it is sufficient to regard the controller as a simple computer. For example, a proportional controller may be thought of as a device that receives the error signal and puts out a signal proportional to it. Similarly, the final control element may be regarded as a device that produces corrective action on the process. The corrective action is regarded as mathematically related to the output signal from the controller. However, it is desirable to have some appreciation of the actual physical mechanisms used to accomplish this. This chapter contains with a physical description of a pneumatic control valve and a simplified description of a proportional controller.

Upto about 1960, most controllers were pneumatic. Although pneumatic controllers are still in use and function quite well in many installations, the controllers being installed today are electronic or computer-based instruments.

MECHANISMS

Control Valve

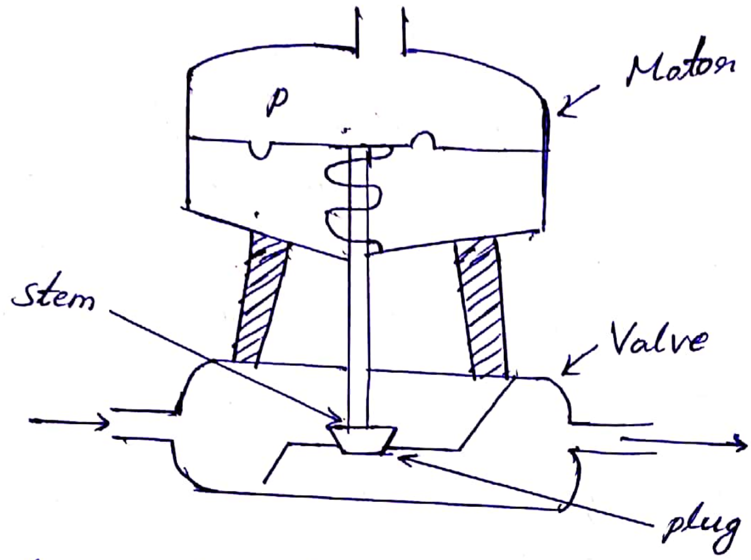


Fig - Pneumatic Control Valve. (air-to-close)

The Control valve shown in fig. contains a pneumatic device (valve motor) that moves the valve stem as the pressure on a spring-loaded diaphragm changes. The stem positions a plug in the orifice of the valve body.

As the pressure increases, the plug moves downward and restricts the flow of fluid through the valve. This action is referred to as air-to-close. The valve can also be constructed to have air-to-open action. Valve motors are often constructed so that the valve stem positions is proportional to the valve-top pressure. Most commercial valves move from fully open to fully closed as the valve-top pressure changes from 3 to 15 psig.

* Fully open → Valve top pressure – 3 psig.
Fully closed → Valve top pressure – 15 psig.

In general, the flow rate of fluid through the valve depends upon the upstream and downstream fluid pressures and the size of the opening through the valve. The plug and seat (or orifice) can be shaped so that various relationships between stem position and size of opening (hence, flow rate) are obtained.

At s.s, the flow (for fixed upstream and downstream fluid pressures) is proportional to the valve-top pneumatic pressure. A valve having this relation is called a linear valve.

Controller

The hardware required for controllers consists of following components with their respective conversions:

1. Transducer (temp. to current).
2. Controller-recorder (current to current).
3. Converter (current to pressure).
4. Control valve (pressure to flow rate).

The control hardware required to control the temp. of a stream leaving a heat exchanger is shown in fig.

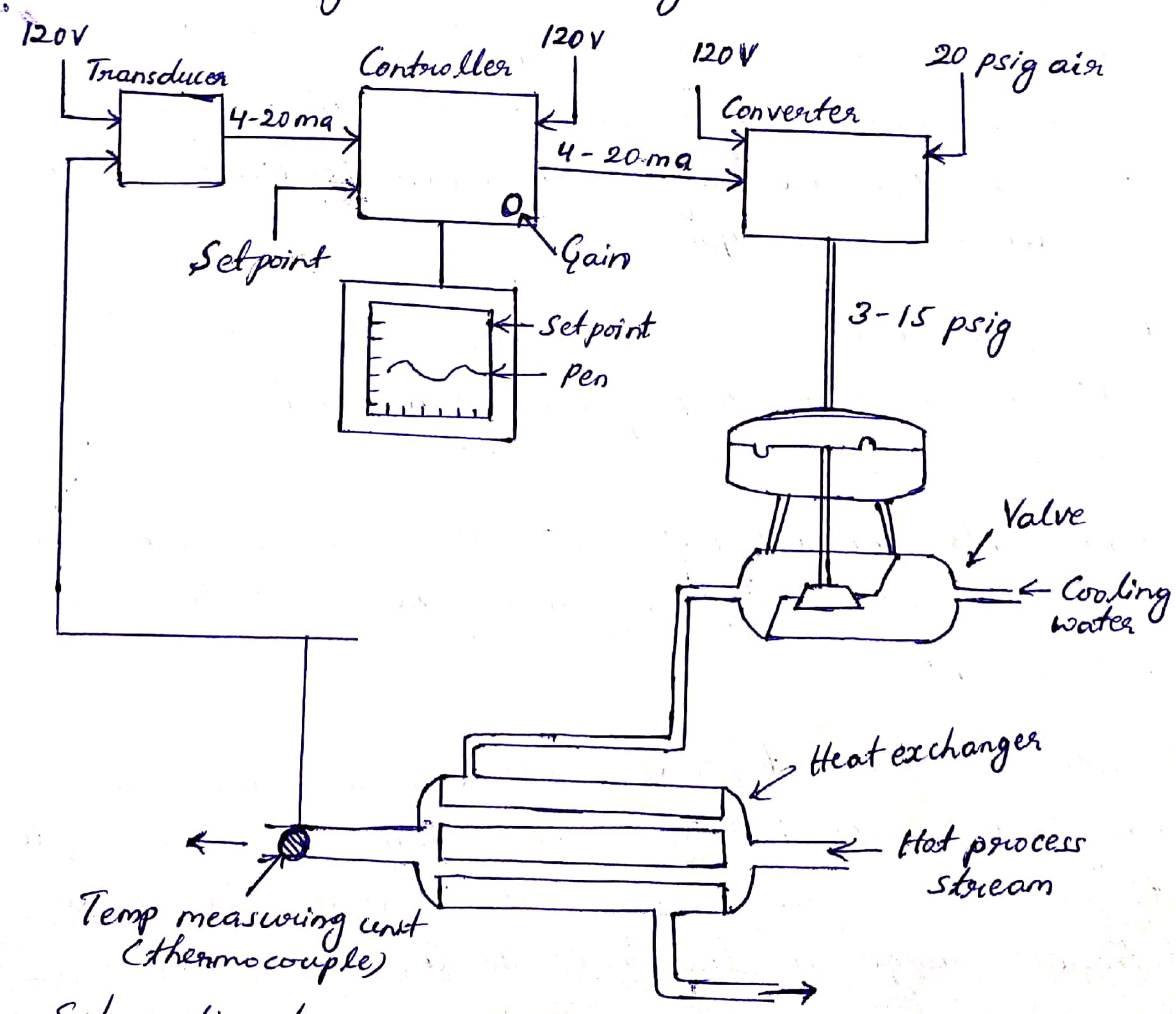


Fig - Schematic diagram of control system

Fig. shows that a thermocouple is used to measure the temp.

* The signal from the thermocouple is sent to the transducer, which produces an output in the range of 4-20 ma, which is a linear function of the input.

* The output of the transducer enters the controller where it is compared to the set point to produce an error signal.

* The controller converts the error to an output in the range of 4-20ma according to the control law stored in the memory of the computer.

~~* The only control law~~

* The output of the controller enters the converter, which produces an output in the range of 3-15 psig, which is a linear function of the input.

* Finally, the output of the converter is sent to the top of the control valve, which adjusts the flow of cooling water to the heat exchanger. (The valve is linear and is the pressure-to-open type.

*

* Electricity is needed for the transducer, controller, and converter. A source of 20 psig air is needed for the converter.

* The components interact with each other, consider the process to be operating at steady state with the outlet temp. equal to the set point.

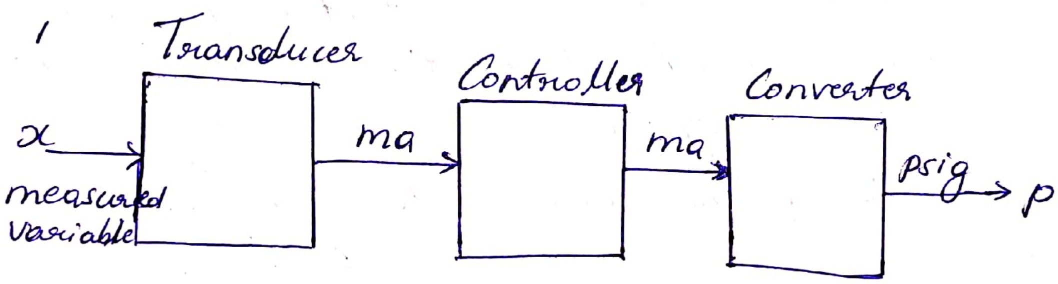
* If the temp. of the hot process stream increases, the following events occur: After some delay the thermocouple detects an increase in the outlet temp. and produces a proportional change in the signal to the controller.

* As soon as the controller detects the rise in temp. relative to the set point, the controller output increases according to proportional action.

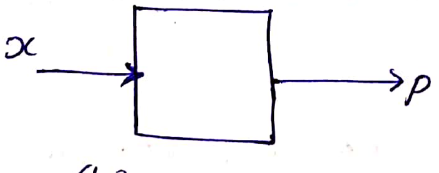
* The increase in signal to the converter causes the output pressure from the converter to increase and open the valve wider in order to admit a greater flow of cooling water.

* The increased flow of cooling water will eventually reduce the output temp. and move it toward the set point. From this qualitative description, the flow of signals from one component to the next is such that the temp. of the heat exchanger should return toward the set point.

* In a well-tuned control system, the response of the temp. will oscillate around the set point before coming to steady state.



(a)



(b)

Fig - Equivalent block transducer, controller and converter

IDEAL TRANSFER FUNCTIONS

Control Valve

A pneumatic valve always has some dynamic lag, which means that the stem motions does not respond instantaneously to a change in the applied pressure from the controller.

The relationship between flow and valve-top pressure for a linear valve is represented by first-order transfer function,

i.e. $$\frac{Q(s)}{P(s)} = \frac{K_V}{\tau_V s+1}$$

where,

K_V - is the s.s gain i.e. constant of proportionality between steady-state flow rate and valve-top pressure, and

τ_V - time constant of the valve.

In many practical systems, the time constant of the valve is very small when compared with the time constants of other components of the control system.

The transfer function of the valve can be approximated by a constant,

$$\therefore \frac{Q(s)}{P(s)} = K_V$$

Under these conditions, the valve is said to contribute negligible dynamic lag.

Consider a first-order valve and a first-order valve and a first-order process connected in series, the transfer function from $P(s)$ to $Y(s)$ is,

$$\frac{Y(s)}{P(s)} = \frac{K_V K_P}{(\tau_V s+1)(\tau_P s+1)}$$

Assumption of no interaction is valid for this case.

For a unit-step change in P,

$$Y = \frac{1}{s} \frac{K_V K_P}{(\tau_V s+1)(\tau_P s+1)}$$

The inverse is,

$$Y(t) = (K_V K_P)\left[1 - \frac{\tau_V \tau_P}{\tau_V - \tau_P}\left(\frac{1}{\tau_P} e^{-t/\tau_V} - \frac{1}{\tau_V} e^{-t/\tau_P}\right)\right]$$

If $\tau_V << \tau_P$, eqn is

$$Y(t) = K_V K_P (1 - e^{-t/\tau_P})$$

The unit-step response of the transfer function.

$$\frac{Y(s)}{P(s)} = K_V \frac{K_P}{\tau_P s + 1}$$

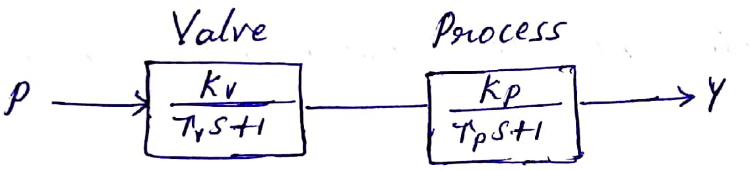


Fig- Block diagram for a first-order valve and a first-order process.

The combination of process and valve is essentially ~~first-ord-~~ first-order. This clearly demonstrates that, when the time constant of the valve is much smaller than that of the process, the valve transfer function can be taken as K_V.

A typical ~~prematic~~ pneumatic valve has a time constant of the order of 1sec. Many industrial processes behave as first-order systems or as a series of first-order systems having time constants that may range from a minute to an hour. For these systems the lag of the valve is negligible.

Controllers

The transfer functions for the controllers frequently used in industrial processes. Because the transducer and converter will be lumped together with the controller for simplicity, the result is that the input will be the measured variable x (ex.- temp, level etc). and output will be a pneumatic signal p. Actually this form (x as input and p as output) applies to a pneumatic controller.

Proportional Control:

The proportional controller produces an output signal (pressure in the case of a pneumatic controller, current or voltage for an electronic controller) that is proportional to the error ϵ.

$$p = K_c \epsilon + p_s \qquad \text{—— (1)}$$

where,

p - output signal from controller, psig or ma

K_c - gain or sensitivity

ϵ - Error = setpoint − measured variable

p_s - a constant.

The error ϵ, which is the difference between the set point and the signal from the measuring element, may be in any suitable units. However, the units of set point and measured variable must be the same, since the error is the difference between these quantities.

In a controller having adjustable gain, the value of the gain K_c can be varied by moving a knob in the controller.

The value of p_s is the value of the output signal when ϵ is zero, and in most controllers p_s can be adjusted to obtain the required output signal when the control system is at steady state and $\epsilon = 0$.

To obtain the transfer function of eq. ①, introduce the deviation variable

$$P = p - p_s$$

At time $t = 0$, assume the error $\epsilon_s = 0$. Then ϵ is already a deviation variable. Eq ① becomes,

$$P(t) = K_c \, \epsilon(t) \quad \text{——— ②}$$

Taking the transform of eq② gives the transfer function of an ideal proportional controller,

$$\frac{P(s)}{\epsilon(s)} = K_c \quad \text{——— ③}$$

Instead of term "gain", "proportional band" is commonly used among process control engineers.

<u>Proportional Band (Pb) or Band Width</u>

It is defined as the error (expressed as a percentage of the range of measured variable). required to move the valve from fully closed to fully open.

Proportional gain corresponds inversely with proportional band. i.e

$$\text{proportional gain} \propto \frac{1}{\text{proportional band}}$$

The gain K_c - has the units psi/unit of measured variable. The relation b/n proportional band in % and K_c. i.e. $K_c = \frac{100}{[Pb\%]}$

If the actual controller is considered, both the input and output units are in milliampers. - In this case the gain will be dimensionless [i.e. ma/ma].

ON-OFF CONTROL

A special case of proportional control is on-off control. If the gain K_c is made very high, the valve will move from one extreme position to the other if the pen deviates only slightly from the set point. This very sensitive action is called on-off action because the valve is either fully open (on) or fully closed (off) i.e. the valve acts like a switch.

Ex - This is a very simple controller and is exemplified by the thermostat used in a home-heating system.

The bandwidth of an on-off controller is approximately zero.

PROPORTIONAL - INTEGRAL (PI) CONTROL :

This mode of control is described by the relationship,

$$p = K_c \epsilon + \frac{K_c}{\tau_I} \int_0^t \epsilon \, dt + p_s \quad \text{——} \; ①$$

where, K_c = gain

τ_I - Integral time, min

p_s - constant

In this case, added to the proportional action term, $K_c \epsilon$, another term is proportional to the integral of the error. The values of K_c and τ_I may be varied by two knobs in the controller.

To visualize the response of this controller, consider the response to a unit-step change in error in fig. This unit-step response is most directly obtained by inserting $\epsilon = 1$ into eq ① which yields

$$p(t) = K_c + \frac{K_c}{\tau_I} t + p_s \quad \text{——} \; ②$$

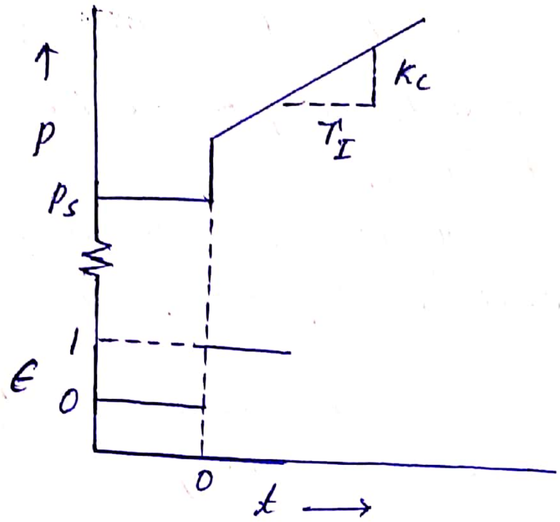


Fig - Response of a PI controller to a unit-step change in error.

Notice that p changes suddenly by an amount K_c, and then changes linearly with time at a rate K_c/τ_I.

To obtain the transfer function of eq ① introduce the deviation variable $P = p - p_s$ into ① and then take the transform to obtain,

$$\frac{P(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) \quad \text{———} \quad ③$$

Reset rate - defined as the reciprocal of τ_I i.e $\frac{1}{\tau_I}$

The integral adjustment knob on a controller may be marked in terms of integral time OR reset rate.

The calibration of the proportional and integral knobs is often checked by observing the jump and slope of the step response shown in fig.

PROPORTIONAL - DERIVATIVE (PD) CONTROL:

This mode of control may be represented by

$$p = K_c \epsilon + K_c \tau_D \frac{d\epsilon}{dt} + p_s \quad \text{———} \quad ①$$

where, K_c - gain

τ_D - derivative time, min

p_s - constant

In this case, proportional derivative term $K_c \tau_D \frac{d\epsilon}{dt}$, added which is proportional to the derivative of the error. The values of K_c and τ_D may be varied separately by knobs on the controller.
Other terms that are used to describe the derivative action are "rate control" and "anticipatory control".

The action of this controller can be visualized by considering the response to a linear change in error as shown in fig.

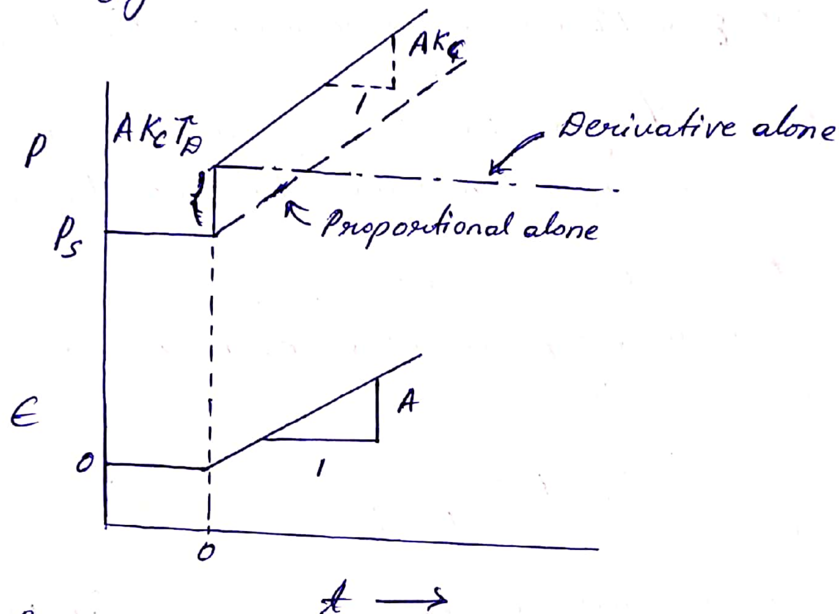


Fig - Response of a PD controller to a linear input in error.

This response is obtained by introducing the linear function $\epsilon(t) = At$ into eq. ① to get,

$$p(t) = AK_c t + AK_c\tau_D + p_s$$

Notice that p changes suddenly by an amount $AK_c\tau_D$ as a result of the derivative action and then changes linearly at a rate AK_c. The effect of derivative action in this case is to anticipate the linear change in error by adding additional output $AK_c\tau_D$ to the proportional action.

To obtain the transfer function from eqn. ① introduce the deviation variable $P = p - p_s$ and then take the transform to obtain

$$\frac{P(s)}{\epsilon(s)} = K_c(1+\tau_D s) \quad \text{—— ②}$$

PROPORTIONAL - INTEGRAL - DERIVATIVE (PID) CONTROL :

This mode of control is combination of the previous modes and is given by the expression.

$$p = K_c \epsilon + K_c \tau_D \frac{d\epsilon}{dt} + \frac{K_c}{\tau_I}\int_0^t \epsilon\, dt + p_s \quad \text{—— ①}$$

In this case, the controller contains three knobs for adjusting K_c, τ_D and τ_I. The transfer function for this controller can be obtained from the Laplace transform of eq. ① thus

$$\frac{P(s)}{\epsilon(s)} = K_c\left(1 + \tau_D s + \frac{1}{\tau_I s}\right) \quad \text{—— ②}$$

Motivation for addition of Integral and Derivative Control modes :

The fig. shown consists of curves showing the behaviour of a typical, feed back control system using different kinds of control when it is subjected to a permanent disturbance.

Ex - Tank temp. control system for a step change in T_i.

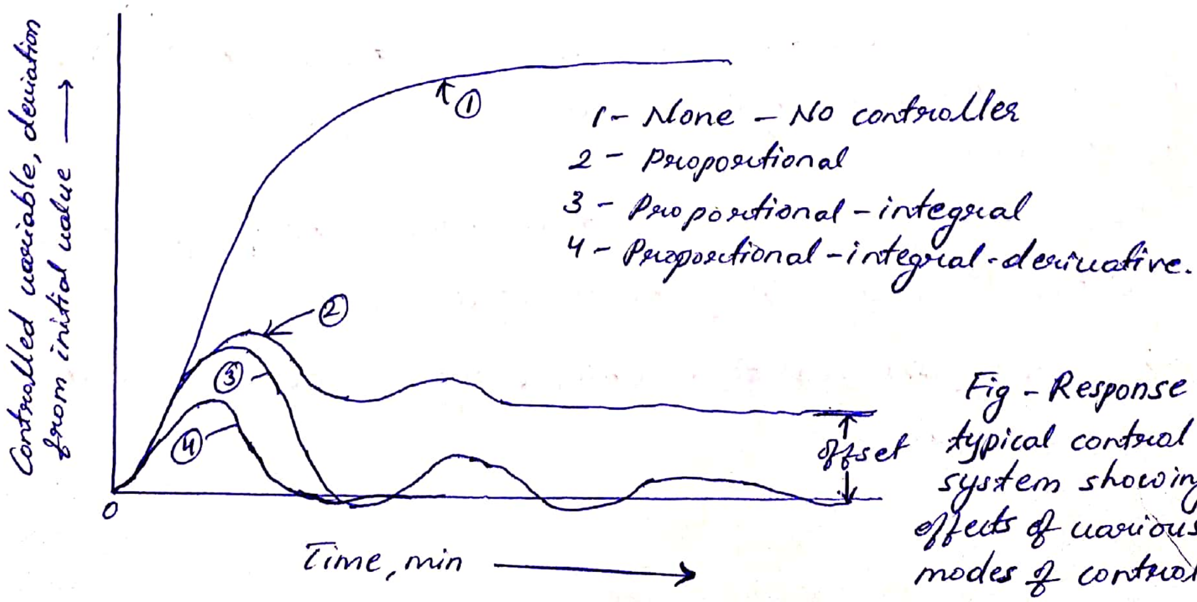


Fig - Response of a typical control system showing the effects of various modes of control

* The value of the controlled variable is seen to rise at time zero owing to the disturbance.
* With no control, this variable continues to rise to a new s.s value.
* With only proportional, the control system is able to arrest the rise of the controlled variable and ultimately bring it to rest at a new s.s. value.

The difference b/n this new s.s value and the original value is called offset.

* For a particular system shown, the offset is seen to be only 22% of the ultimate change that would have been realized for this disturbance in the absence of control.
* As shown by PI curve, the addition of integral action eliminates the offset; the controlled variable ultimately returns to the original value.
* This advantage of integral action is balanced by the disadvantage of a more oscillatory behavior.
* The addition of derivative action to the PI action gives a definite improvement in the response. The rise of the controlled variable is arrested more quickly and it is returned rapidly to the original value with the little or no oscillation.
* The selection among the control systems whose responses are as shown in fig. depends on the particular application.
  → If an offset of 22% is tolerable, then proportional action would likely be selected.
  → If no offset is tolerable, integral action would be added.

→ If excessive oscillations had to be eliminated, derivative action might be added.

* The addition of each mode indicates that more difficult controller adjustment.

CONTROL VALVES

Basic components of any control system - is the final control element, which comes in a variety of forms depending on the specific control application.

The most common type of final control element in chemical processing is the pneumatic control valve, which regulates the flow of fluids.

Other examples - Variable speed pump
Power controller (used in electrical heating)

Control Valve Construction:

The control valve is essentially a variable resistance to the flow of a fluid, in which the resistance and therefore the flow, can be changed by a signal from a process controller.

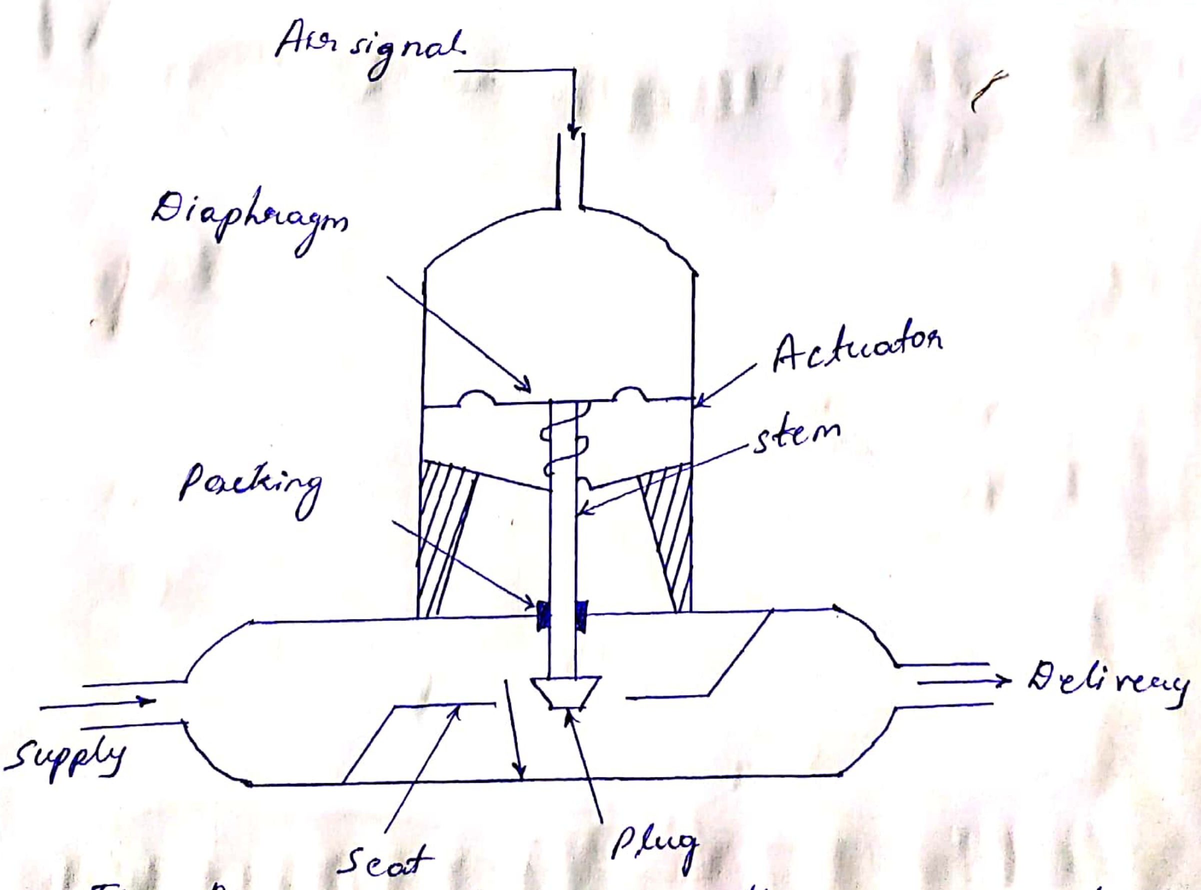


Fig - Pressure-to-close valve with ~~signal~~ single seating.

* The control valve consists of an actuator and a valve. The valve itself is divided into the body and the trim.

Body - It consists of a housing for mounting the actuator and connections for attachment of the valve to a supply line and a delivery line.

Trim - The trim, which is enclosed within the body, consists of a plug, a valve seat, and a valve stem.

* The actuator moves the valve stem as the pressure on a spring-loaded diaphragm changes.
* The stem moves ~~the valve~~ a plug in a valve seat in order to change the resistance to flow through the valve.
* When a valve is supplied by the manufacturer, the actuator and the valve are attached to each other to form one unit.
* For most actuators, the motion of the stem is proportional to the pressure applied on the diaphragm. This type of actuator can be used for functions other than moving a valve stem.

  Ex - It can be used to adjust dampers, variable speed drives, rheostats, etc.

* As pressure to the valve varies over its normal range of operation (3 to 15 psig) the range of motion of the stem varies from a fraction of an inch of to several inches depending on the size of the actuator. Manufacturers provide a range of actuators for various valve sizes.

Valve Sizes -

The valves available vary over a wide range of sizes. The size is usually referred to by the size of the end connectors.

Ex - A one-inch valve would have connectors (threaded or flanged) to fit into a one-inch pipe line.

In general, the larger the valve size the larger the flow capacity. of the valve.

Working

An increase in signal pressure above the diaphragm exerts a force on the diaphragm and back plate, which causes the stem to move down.

* This causes the cross-sectional area for flow between the plug and the seat to decrease, thereby reducing or throttling the flow. Such a valve action as shown in fig. is called pressure-to-close action.
* The reverse action, pressure-to-open, can be accomplished by designing the actuator so that pressure is applied to the under side of the diaphragm, for which case an increase in pressure to the valve raises the stem.

In this the plug is inverted on the stem and placed under valve seat. This action is called pressure-to-open.

Single-seated Valve:

It contains one plug with one seating surface. For a single-seated valve, the plug must open against the full pressure drop across the valve. If the pressure drop is large, this means that a larger, more expensive

To overcome this problem, valves are also constructed with double seating as shown in fig. below. If tight shut is required, then single seated valves are used.

Double-Seated Valve:

In this type valve, two plugs are attached to the valve stem and each one has a seat.

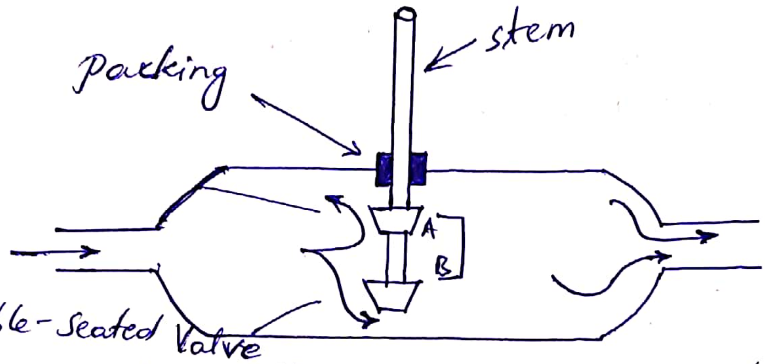


Fig - Double-Seated Valve

The flow pattern through the valve is designed so that the pressure drop across the seat at A tends to open ~~the valve is designed so that~~ the plug and the pressure drop across the seat at B tends to close the plug. This counterbalancing of forces on the plugs reduces the effort needed to open the valve with the result that a smaller, less expensive actuator is needed.

In a double-seated valve, it is difficult to have tight shut-off. If one plug has tight closure, there is usually a smaller gap between the ~~often~~ other plug and its seat.

In many processes, the valve is used for throttling flow and is never expected to operate near its shut-off position. For these conditions, the fact that the valve has a small leakage at shut-off position does not create a problem.

VALVE SIZING:

The size of a valve in terms of its capacity to provide flow when fully open,

$$q = C_V\sqrt{\frac{\Delta P_V}{G}} \quad \text{or} \quad q = K_V\sqrt{\frac{\Delta P_V}{G}} \qquad \text{—— (1)}$$

where, q - flow rate, gpm , m^3/hr (S.I)

ΔP_V - pressure drop across the wide-open valve, kgf/cm^2

G - sp. gravity of fluid at stream temp relative in water. for water $G = 1$

C_V - factor associated with capacity of valve.

Eq. applies to the flow of an incompressible fluid through a fully open valve. Manufacturers rate the size of a valve in terms of the factor C_V.

C_V - is defined as the flow (gpm) of a fluid of unit specific gravity through a fully open valve, across which a pressure drop of 1.0 lbf/in^2 exists.

i.e. $q = 1$, $\Delta P_V = 1$ and $G = 1$

$K_V = 0.856\ C_V$

Eq. (1) is based on the well-known Bernoulli eq. for determining the pressure drop across valves and resistances. It is important to emphasize that C_V is determined using the units listed.

For gases and steam, modified versions are used in which C_V is still used as a factor. Manufacturers of valves provide brochures, nomographs, and special slide rules for sizing valves for use with gases and steam.

In general, as the physical size of a valve body (i.e. size of pipe connectors) increases, the value of C_v increases.
For a sliding stem and plug type of control valve, the value of C_v is roughly equal to the square of the pipe size multiplied by ten.

i.e. $C_v \cong (\text{pipe size})^2 \times 10$ for 3"

Example - For 3" control valve should have C_v of about 90

i.e. $C_v = (3)^2 \times 10 = 90$

$C_v = f(\text{shape, size, roughness, } Re)$

Example 1 - A valve with a C_v rating of 4.0 is used to throttle the flow of glycerine for which $G = 1.26$. Determine the maximum flow through the valve for a pressure drop of 100 psi.

→

$$q = C_v \sqrt{\frac{\Delta p_v}{G}}$$

$$q = 4.0\sqrt{\frac{100}{1.26}} = 35.6 \text{ gpm}$$

The coefficient C_v varies with the design of the valve (shape, size, roughness) and the Reynolds number for the flow through the valve. This relationship is analogous to the relationship between friction factor and roughness and Reynolds number for flow through a pipe. For relatively non-viscous fluids C_v in Eq (1) can be taken as a constant for a valve of given size and type. The reason for this is that at high Reynolds nos., the friction factor changes very little with flow rate. Except for very viscous fluids, the flow through a valve, which involves sudden contraction and expansion, is in the turbulent regime of fluid flow; turbulence in the valve exists even if the flow in the supply pipe is near the critical Reynolds number of 2100.

Consequently, for relatively nonviscous fluids, eq ① is satisfactory for sizing a valve for any fluid. For the control of flow of very viscous fluids, such as tar or molasses, the value of C_v found from eq ① must be multiplied by a correction factor that depends on viscosity density, flow rate and valve size (i.e. on the Reynolds no.) Methods for determining the viscosity correction factor are provided by manufacturers for their valves. If one does not apply the correction factor for a very viscous fluid, the value of C_v will be too low and the valve will be undersized.

Valve Characteristics :

The function of a control valve is to vary the flow of fluid through the valve by means of a change of pressure to the valve top.

The relation between the flow through the valve and the valve stem position (or lift) is called the valve characteristic,

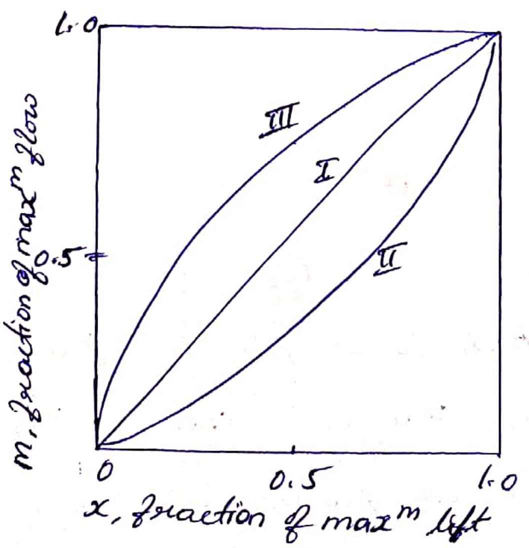


Fig - Inherent valve characteristics (pressure drop across valve is costant)

I - Linear

II - Increasing sensitivity (e.g. equal percentage valve)

III - Decreasing sensitivity

In general, the flow through a control valve for a specific fluid at a given temp. can be expressed as;

$$q = f_1(L, p_0, p_1) \quad \text{—— (2)}$$

where,

q - volumetric flow rate

L - valve stem position (or lift)

p_0 - upstream pressure

p_1 - downstream pressure

The inherent valve characteristic is determined for fixed values of p_0 and p_1 for which case, eq (2) becomes,

$$q = f_2(L) \quad \text{—— (3)}$$

For convenience let:

$$m = \frac{q}{q_{max}} \quad \text{and} \quad x = \frac{L}{L_{max}}$$

where, q_{max} - is the maximum flow when the valve stem is at its maximum lift

L_{max} - valve is full-open

x - is the fraction of maximum lift

m - is the fraction of maximum flow.

Eqn (3) may be written as,

$$m = \frac{q}{q_{max}} = f\left(\frac{L}{L_{max}}\right)$$

(or)

$$m = f(x) \quad \text{—— (4)}$$

The types of valve characteristics can be defined in terms of the sensitivity of the valve, which is simply the fractional change in flow to the fractional change in stem position for fixed upstream and downstream pressures.

Mathematically,

$$\text{sensitivity} = \frac{dm}{dx}$$

In terms of valve characteristics, valves can be divided into three types:
Decreasing sensitivity, linear and increasing sensitivity.

* For the decreasing sensitivity type, the sensitivity (or slope) decreases with m.
* For the linear type, the sensitivity is constant and the characteristic curve is a straight line.
* For the increasing sensitivity type, the sensitivity increases with flow.

Valve characteristics curves, as in fig. can be obtained experimentally for any valve by measuring the flow through the valve as a function of lift (or valve-top pressure) under conditions of constant upstream and downstream pressures.

Two types of valves that are widely used are

1. Linear valve - is one for which the sensitivity is constant and the relation between flow and lift is linear.

2. Equal percentage valve (Logarithmic valve) - is of the increasing sensitivity type

Mathematical expression for the linear valve :

For linear valve,

$$\frac{dm}{dx} = \alpha. \quad -①$$

where, α – is a constant

Assuming that the valve is shut tight when the lift is at lowest position, i.e. $m=0$ at $x=0$.

For a single-seated valve that is not badly worn, the valve can be shut off for $x=0$.

On integrating eq. ① and introducing the limits

At $x=0$, $m=0$ and.

at $x=1$, $m=1$ gives,

$$\int_0^1 dm = \int_0^1 \alpha \, dx$$

Integrating this eq. and inserting limits gives

$$m = x \quad \text{(linear valve)} \; —— ②$$

Mathematical expression for equal percentage valve

For equal percentage valve,

$$\frac{dm}{dx} = \beta m \; —— ①$$

where, β – is constant.

On integration,

$$\int_{m_0}^{m} \frac{dm}{m} = \int_0^x \beta \, dx$$

$$\ln \frac{m}{m_0} = \beta x \; —— ②$$

where, m_0 – is the flow at $x=0$.

A plot of m V/s x on semi-log paper gives a straight line. Eq. ② is the basis for calling the valve characteristic logarithmic.

On rearranging eq. ①

$$\frac{dm}{m} = \beta\, dx.$$

or

$$\frac{\Delta m}{m} = \beta\, \Delta x$$

In this form it can be seen that an equal fractional (or percentage) change in flow ($\Delta m/m$) occurs for a specified increment of change in stem position (Δx), regardless of where the change in stem position occurs along the characteristic curve.

The term β can be expressed in terms of m_0 by inserting $m=1$ at $x=1$ into eq. ②. The result is,

$$\beta = \ln\left(\frac{1}{m_0}\right)$$

Solving eq. ② for m gives,

$$m = m_0 e^{\beta x} \text{ (equal \% valve)} \quad \text{——} \ ③$$

In ~~t~~ integrating eq ①, the flow was assumed to be m_0 at $x=0$. This is mathematically necessary, because m_0 cannot be taken as zero at $x=0$ because as LHS term in eq. ② becomes infinite.

In practice, there may be some leakage (hence $m_0 \neq 0$) when the stem is at its lowest position for a double-seated valve or for a valve in which the plug and seat have become worn.

For some valves, especially large ones, the valve manufacturer intentionally allows some leakage at minimum left ($x=0$) to prevent binding and wearing of the plug and seat surfaces. For a valve that does shut tight and is also classified as an equal percentage valve, the equal percentage characteristic will not be followed when the valve is nearly shut. In practice, the control valve serves as a throttling valve and is not intended to be wide-open or completely closed during normal operation.

In order to express the range over which an equal percentage valve will follow the equal percentage characteristic; the term rangeability is used.

Rangeability – It is defined as the ratio of maximum flow to minimum controllable flow over which the valve characteristic is followed.

$$\text{Rangeability} = \frac{m_{max}}{m_{min,\,controllable}}$$

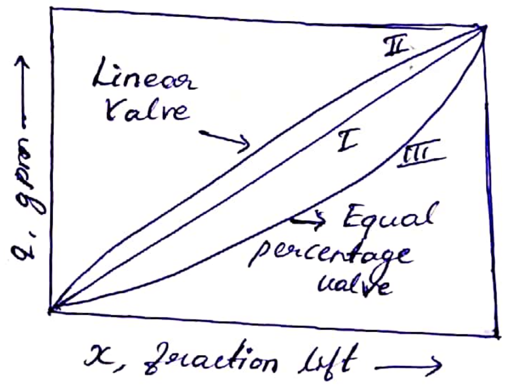
For example – If m_o is 0.02, the rangeability is 50.

Effective Valve Characteristic

When a valve is placed in a line that offers resistance to flow, the inherent characteristic of the valve will be altered. The relation between flow and stem position (or valve-top pressure) for a valve installed in a process line will be called the "Effective valve characteristic".

Benefit of an equal percentage valve:

Benefit derived from an equal percentage valve arises from its inherent non-linear characteristic that compensates for the line loss to give an effective valve characteristic that is nearly linear.



An equal percentage valve overcompensates for line loss and produces an effective characteristic that is not linear, but is bowed in the opposite direction to that of the effective characteristic of the linear valve.

In general, neither valve produces an effective characteristic that is linear. The line loss increases, the linear valve will depart more from the ideal linear relation and the equal percentage valve will move more closely toward the linear relation.

In practice, a valve designated as linear will not give a linear characteristic exactly as defined. To achieve a truly linear characteristic would require very careful design and precision machining of the valve plug and seat. In order to know the effective characteristic of a valve, one must test it experimentally.

Valve Positioner:

The friction in the packing and guiding surfaces of a control valve causes a control valve to exhibit hysteresis as in fig. in which stem position is plotted against valve top-pressure.

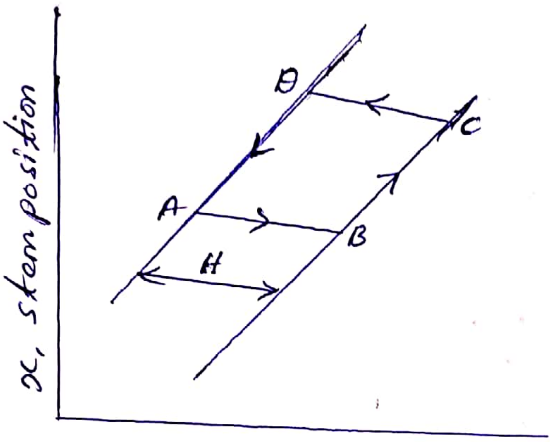


Fig- Control valve hysteresis

When the pressure increases, the stem position increases along the lower curve. When the pressure decreases, the stem position decreases along the upper ~~valve~~ curve. At the moment the air pressure signal reverses, the stem position stays in the last position until the dead H is exceeded, after which the pressure begins to decrease or increase along the paths shown by the arrows. If the valve is subjected to a slow periodic variation in pressure, a typical path taken by the stem position is shown by the closed curve ABCDA in fig.

The dynamic lag is caused by the volume of space above the valve ~~diagram~~ diaphragm, the resistance to flow of air to the valve top, and the inertia of the valve stem and plug; such a lag is expressed by a first-order or second-order transfer function. On the other hand, hysteresis, which is caused by the friction between the stem and the packing, is a non-linear phenomenon and cannot be expressed by a transfer function. A valve can exhibit both dynamic lag and hysteresis.

The presence of hysteresis in the valve can cause the controlled signal to exhibit an oscillations or ripple called a "limit cycle." Since this limit cycle is usually considered objectionable and contributes to wear of the valve, a method is needed to eliminate it. Since the limit cycle is a nonlinear phenomenon related to the hysteresis, controller tuning is not a solution to the problem.

To reduce the deleterious effect of hysteresis and to also speed up the response of the valve, one can attach to the control valve a positioner which acts as a high-gain proportional controller that receives a set-point signal from the primary controller and a measurement from the valve stem position. In this sense, the addition of a valve positioner introduces a form of cascade control. The positioner, bolted to the valve actuator, has an arm that is clamped to the valve stem to detect the stem position.

The valve positioner has the usual connections for a controller: a set point that calls for a desired stem position in the form of a signal from the primary controller p_c, a measurement in the form of stem position x, and a a pneumatic output in the form of a pressure to the valve top P_V. The mechanical details of an actual valve positioner involve a pneumatic mechanism functioning as a high-gain proportional controller. The gain is built into the design of the positioner and cann't be adjusted. The valve positioner is especially important for speeding up the valve motion, and eliminating hysteresis and valve stem position.

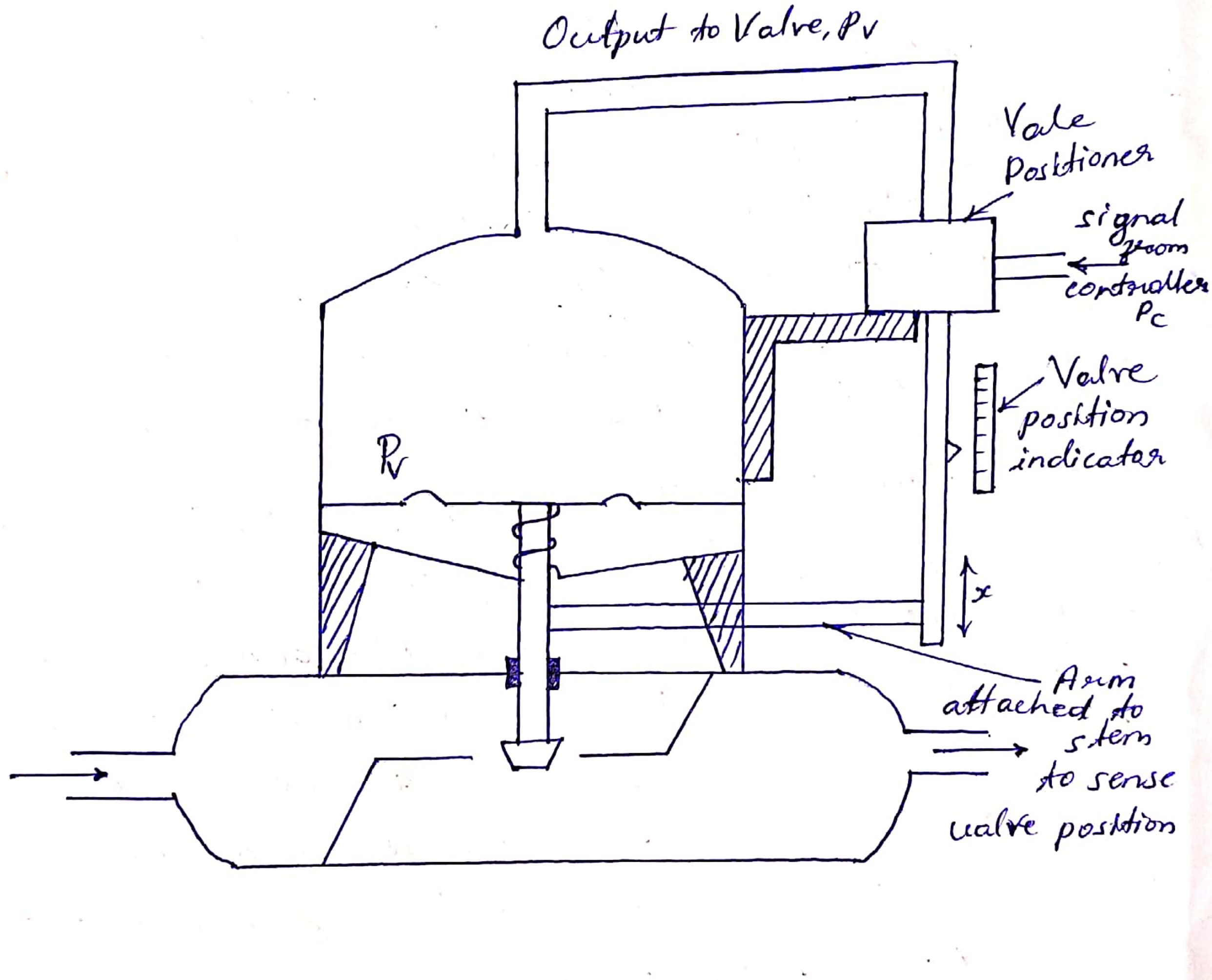


Fig - Control Valve with positioner

CLOSED-LOOP TRANSFER FUNCTIONS

Standard Block-Diagram Symbols

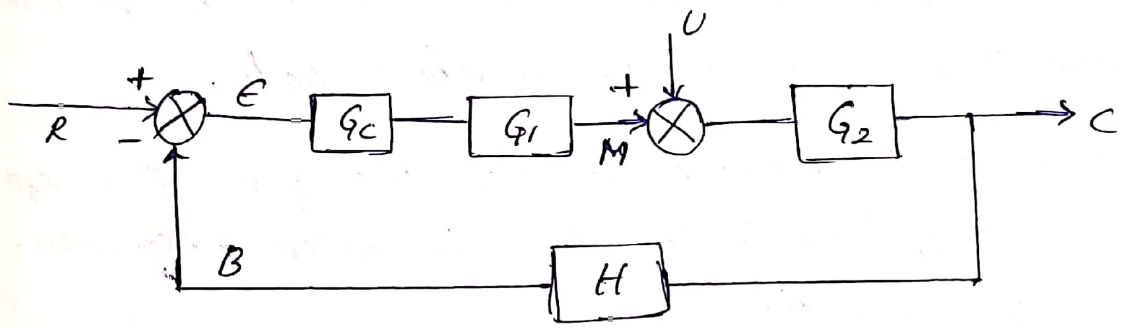


Fig - Standard control system nomenclature.

The block diagram has been incorporates some standard symbols for the variables and transfer functions, which are widely used in the control literature.
These symbols are defined as,

R - set point (OR) desired value
C - controlled variable
E - Error
B - variable produced by measuring element
M - manipulated variable
U - load variable (OR) disturbance
G_c - transfer function of controller
G_1 - transfer function of final control element
G_2 - transfer function of process
H - transfer function of measuring element

* Sometimes, blocks G_c and G_1 will be lumped together into a single block.

* The series of blocks between the comparator and the controlled variable, which consist of G_c, G_1 and G_2 is referred to as the "forward path".

* The block H between the controlled variable and the comparator is called the "feedback path."
* The use of G for a transfer function in the forward path and H for one in the feedback path.
* The product GH, which is the product of all transfer functions ($G_c G_1 G_2 H$) in the loop, is called the open-loop transfer function.

GH open-loop transfer functions because it relates the measured variable B to the set point R if the feedback loop. is ~~called the open-loop transfer function.~~ disconnected (i.e. opened) from the comparator. If in case the variables R and B are connected. in the loop then GH it is closed loop transfer functions.

Overall Transfer function for Single-Loop Systems:

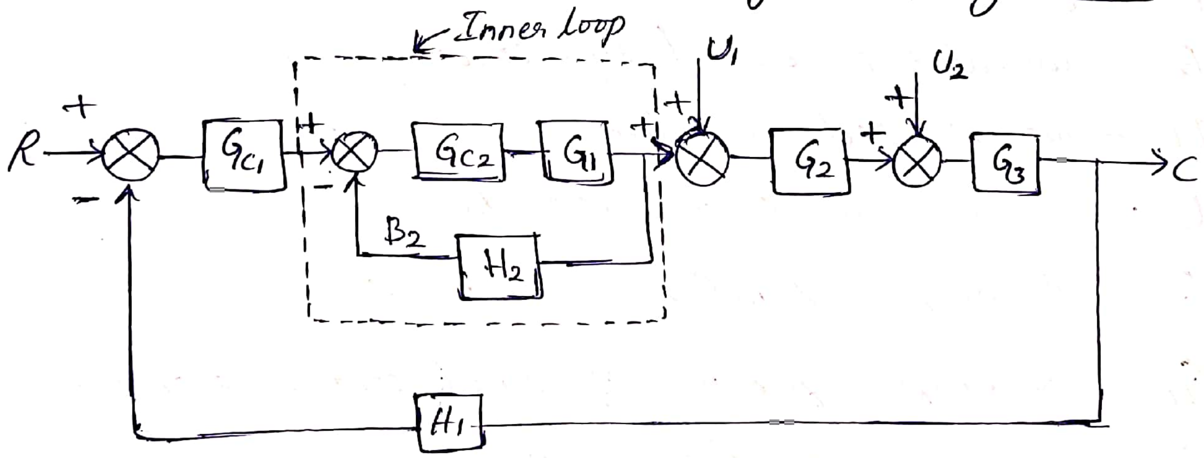


Fig - Block diagram for a multiloop, multiload system

In the previous fig. of Block diagram, the T.F relating C to R or C to U are referred as overall transfer functions because they apply to the entire system.

Eg - For multiloop, multiload system.

Scanned with CamScanner

The transfer functions are useful to determine the response of C to any change in R and U.
The response to a change in set point R, obtained by setting U=0, represents the solution to the servo problem.
The response to a change in load variable U, obtained by setting R=0, is the solution to the regulator problem.

Overall Transfer Function for Change in Set Point

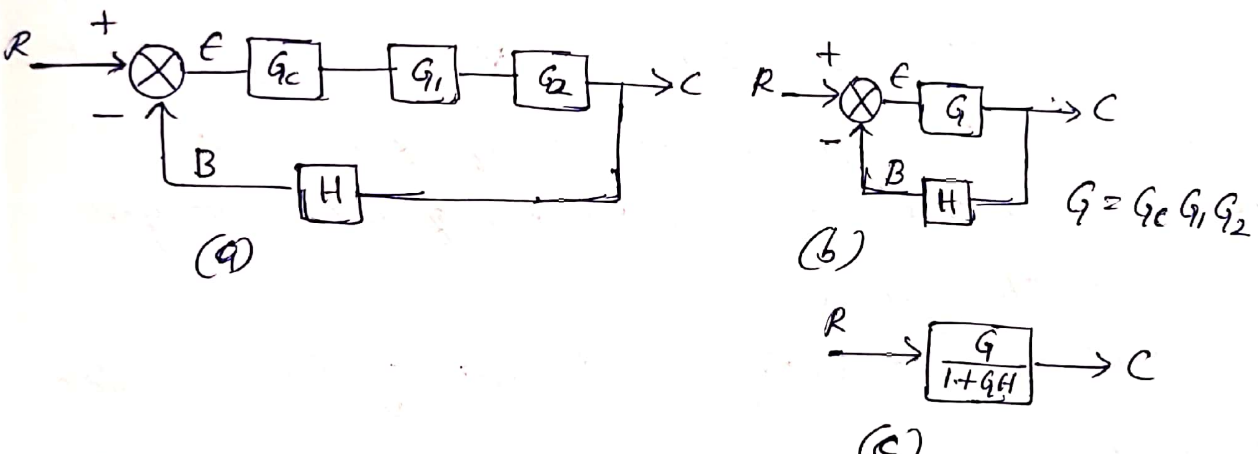


Fig - Block-diagram reduction to obtain overall transfer function.

In this case, $U=0$, Here made use of a simple rule of block-diagram reduction which states that a block diagram consisting of several transfer functions in series can be simplified to a single block containing a transfer functions that is the product of the individual transfer functions.

Proof - Consider two non-interacting blocks in series as shown in fig.

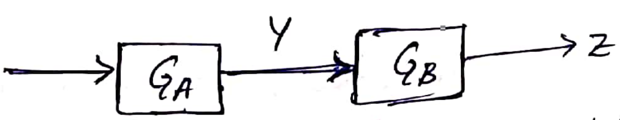


Fig - Two non-interacting blocks in series.

This block diagram is equivalent to the eqns.

$$\frac{Y}{X} = G_A \quad , \quad \frac{Z}{Y} = G_B$$

Multiplying these eqns. gives.

$$\frac{Y}{X} * \frac{Z}{Y} = G_A \cdot G_B$$

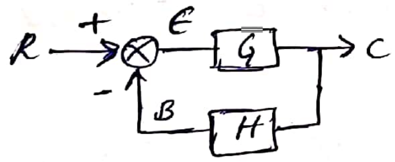
which simplifies to,

$$\frac{Z}{X} = G_A \cdot G_B$$

Thus, the intermediate variable Y has been eliminated, and the overall transfer function Z/X is the product of transfer functions $G_A \cdot G_B$. This proof for two blocks can be easily extended to any number of blocks to give the rule for the general case.

Ex - Several non-interacting first-order system in series.

From fig. (b)



$$C = G E$$
$$B = H C$$
$$E = R - B$$

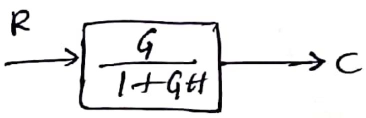
Since there are four variables and three equations, solving the eqns. simultaneously for C in terms of R

$$C = G(R - B)$$
$$C = G(R - HC)$$
$$C = GR - GHC \Rightarrow GR = C(1 + GH)$$

or

$$\frac{C}{R} = \frac{G}{1+GH}$$

This is the overall transfer function relating C to R and is represented by an equivalent block diagram.



Overall Transfer Function for Change in Load

In this case $R=0$.

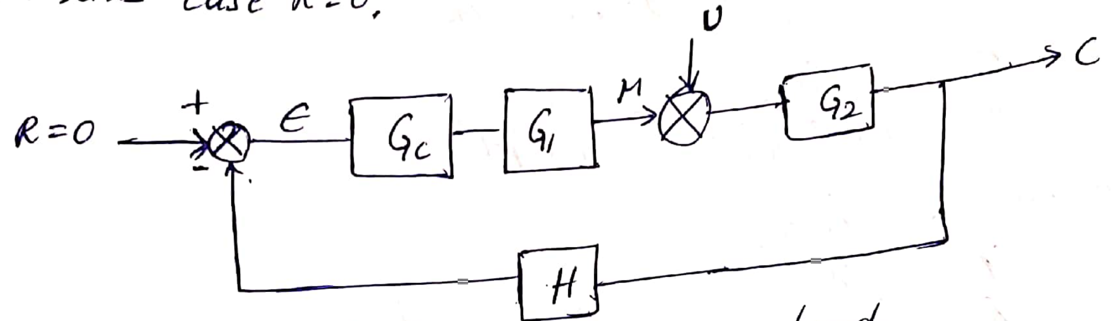


Fig - Block diagram for change in load

From the diagram,

$$C = G_2(U+M)$$

$$M = G_c G_1 \epsilon$$

$$B = HC$$

$$\epsilon = -B$$

Again the number of variables $[C, U, M, B, \epsilon]$ exceeds by one the no. of equations, and solving for C in terms of U.

~~$\epsilon = G_2 (U+M)$~~ $C = G_2[U + G_c G_1 \epsilon]$

~~$M = G_c G_1 \epsilon$~~ $C = G_2[U + G_c G_1(-HC)]$

$$C = G_2 U - G_2 G_c G_1 HC$$

$$C + G_2 G_c G_1 HC = G_2 U$$

$$C[1 + G_2 G_1 G_c H] = G_2 U$$

$$\frac{C}{U} = \frac{G_2}{1+GH}$$

where, $G = G_c G_1 G_2$. The transfer functions for load change or set-point change have denominators that are identical, $1+GH$.

General T.F, $\frac{Y}{X} = \frac{\pi_f}{1+\pi_l}$ — negative feedback.

where,
π_f - product of transfer functions in the path between the locations of the signals X and Y

π_l - product of all transfer functions in the loop

i.e. $\pi_l = G_c G_1 G_2 H$.

If this rule is applied. to find C/R

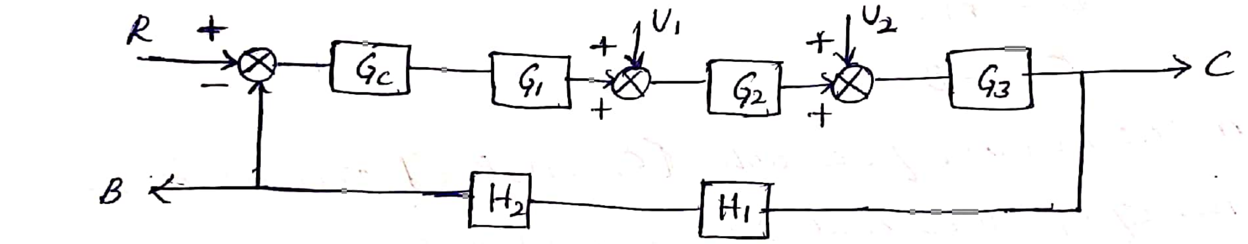
$$\frac{C}{R} = \frac{G_c G_1 G_2}{1 + G_c G_1 G_2 H} = \frac{G}{1+GH}$$

For positive feedback,

$$\frac{Y}{X} = \frac{\pi_f}{1 - \pi_l} \quad \text{positive feedback}$$

Example - Determine the transfer functions C/R, C/U_1 and B/U_2 for the system as in fig. Also determine an expression for C in terms of R and U_1 for the situation when both set-point change and load change occur simultaneously. Use general rule.

→



Using the rule given by. $\frac{Y}{X} = \frac{\pi_f}{1+\pi_l}$

$$\frac{C}{R} = \frac{G_c G_1 G_2 G_3}{1+G} \Rightarrow C = \frac{G_c G_1 G_2 G_3}{1+G} \cdot R$$

$$\frac{C}{U_1} = \frac{G_2 G_3}{1+G} \qquad C = \frac{G_2 G_3}{1+G} U_1$$

$$\frac{B}{U_2} = \frac{G_3 H_1 H_2}{1+G}$$

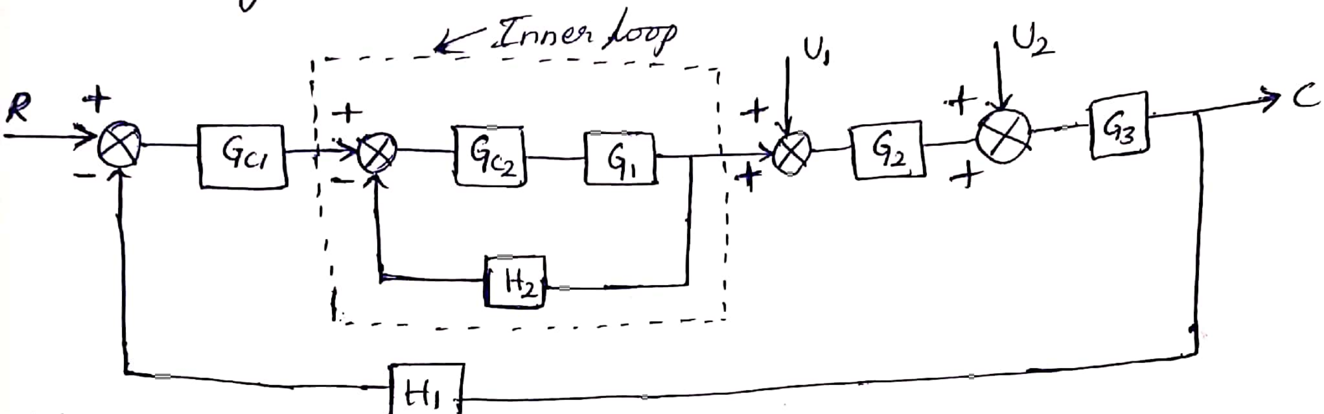
where, $G = G_c G_1 G_2 G_3 H_1 H_2$.

If both R and U_1 occur simultaneously, the principle of superposition requires that the overall response be the sum of the individual responses; thus

$$C = \frac{G_c G_1 G_2 G_3}{1+G} R + \frac{G_2 G_3}{1+G} U_1$$

Overall Transfer Function for Multiloop Control Systems :

Ex- Determine the transfer function C/R for the system shown in fig. This block diagram represents a cascade control system, which will be solved below.



Block diagram reduction

Fig (a) Original diagram

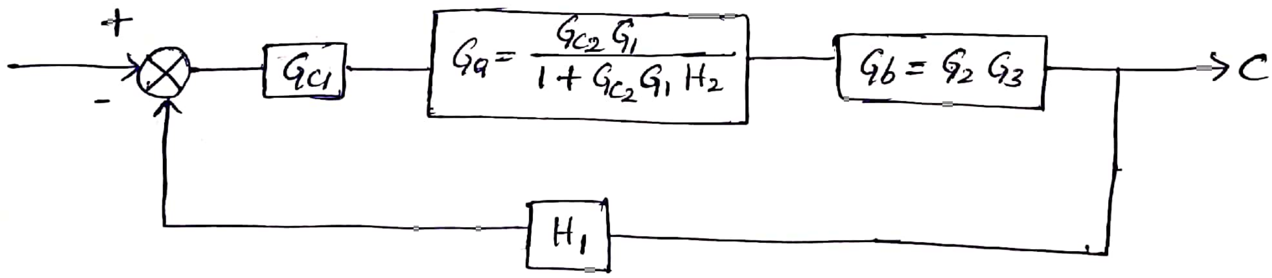


Fig (b) First reduction.

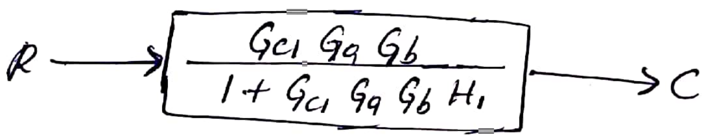
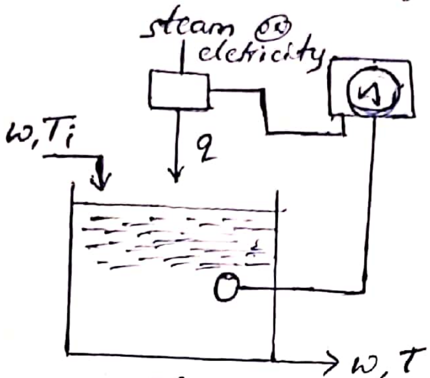


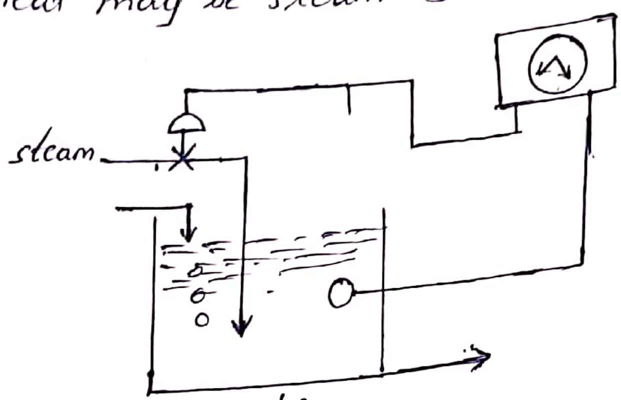
Fig (c) Final single-block diagram

TRANSIENT RESPONSE OF SIMPLE CONTROL SYSTEMS

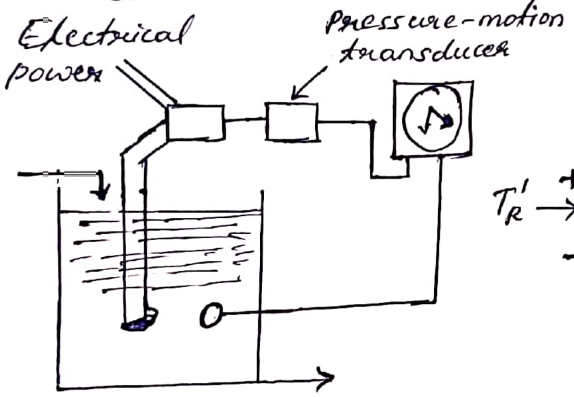
Consider the control system for the heated, stirred tank.
Here the source of heat may be steam or electricity.



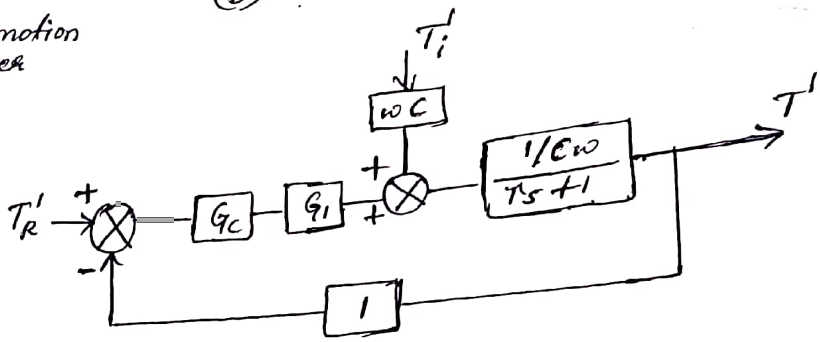
(a)



(b)

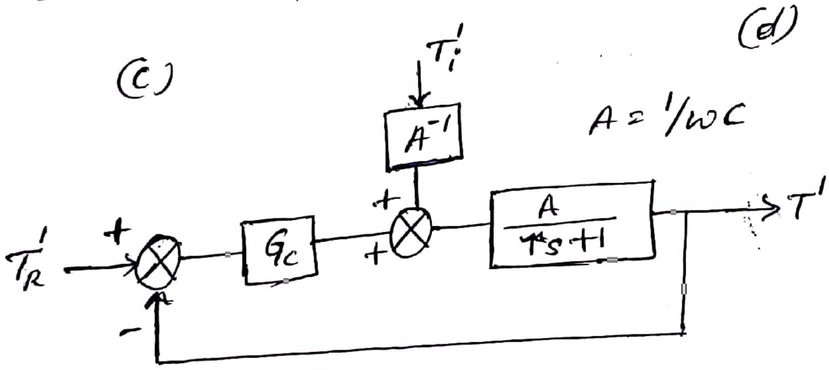


(c)



(d)

$A = 1/wC$



(e)

Fig ~~(e)~~ Block diagram of temperature-control system.

Fig (a) – Source of heat is steam or electricity is not specified.
(b) – Source of heat is steam i.e. discharged directly into water.
(c) – source of heat is electrical
(d) – A device known as a power controller provides electrical power to a resistance heater proportional to the signal from the controller.
(e) – Simplified form of (d) – To reduce the number of symbols of $1/wC$ has been replaced by A.

Assume that the valve does not have any dynamic lag, for which case the transfer function of the valve (G_1) in fig. is taken as a constant K_v. Then further for simplifying steps.

Let $K_v = 1$, If $K_v \neq 1$ then replace G_c by K_c

A bare thermocouple will have a response that is so fast that for all practical purposes it can be assumed to follow the slowly changing bath temperature without lag. When the feedback transfer function is unity. the system is called a unity-feedback system.

Proportional Control for Set-Point Change. (Servo Problem) ($T_i' = 0$)

For proportional control, $G_c = K_c$. The overall transfer function in fig. (e). is $T_i' = 0$

$$\frac{T'}{T_R'} = \frac{K_c A/(\tau s + 1)}{1 + K_c A/(\tau s + 1)} = \frac{K_c A}{\tau s + 1 + K_c A} \quad \text{—— (1)}$$

On rearranging in the form of a first-order lag to give

$$\frac{T'}{T_R'} = \frac{A_1}{\tau_1 s + 1} \quad \text{—— (2)}$$

where, $\tau_1 = \dfrac{\tau}{1 + K_c A}$, $A_1 = \dfrac{K_c A}{1 + K_c A} = \dfrac{1}{1 + 1/K_c A}$

The response of the tank temp. to change in set point is first-order. The time constant for the control system, τ_1, is less than that of the stirred tank itself, τ.
One of the effects of feedback control is to speed up the response.

W.K.T. the response of the system to a unit-step change in set point T_R' is as in fig.

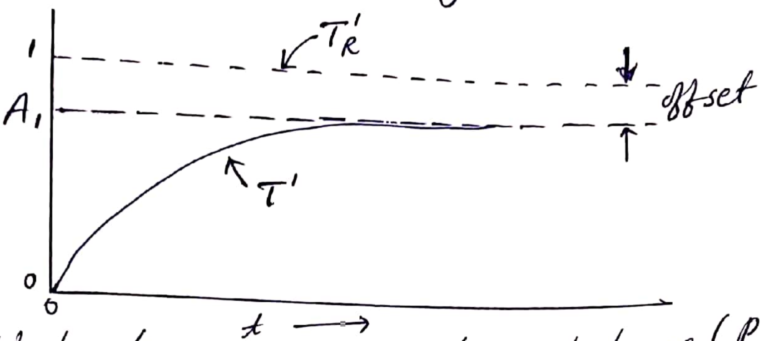


Fig - Unit-step response for set-point change (P control)

For a unit step change in set point, T' approaches A_1

$$A_1 = \frac{K_c A}{(1+K_c A)} \quad \text{, a fraction of unity.}$$

The desired change is, of course 1. Thus, the ultimate value of the temp. $T'(\infty)$ does not match the desired change. This discrepancy is called offset and is defined as

$$\text{offset} = T_R'(\infty) - T'(\infty) \quad \text{——③}$$

In terms of the particular control system parameters.

$$\text{offset} = 1 - \frac{K_c A}{1+K_c A} = \frac{1}{1+K_c A} \quad \text{——④}$$

This discrepancy between setpoint and tank temp. at steady state is characteristic of proportional control. In some cases offset cannot be tolerated. From eq ④ offset decreases as K_c increases, and theoretically offset can be decreased to very small by increasing K_c to large value.

Increasing K_c may lead system to become unstable.

For the present case of proportional control is satisfactory or not depends on the amounts of offset that can be tolerated, the speed of response of the system, and the amount of gain that can be provided by the controller without causing the system to go unstable.

Proportional Control for Load Change (Regulator Problem)

For this case, set point remains constant i.e. $T_R' = 0$.
For a change in inlet stream temp. i.e to a load change

$$\frac{T'}{T_i'} = \frac{A A^{-1}/(\tau s+1)}{1+K_cA/(\tau s+1)} = \frac{1}{\tau s+1+K_cA} \quad \text{——} \quad (1)$$

On rearranging in the form of first-order lag.

$$\frac{T'}{T_i'} = \frac{A_2}{\tau_1 s+1}$$

where, $A_2 = \frac{1}{1+K_cA}$

$$\tau_1 = \frac{\tau}{1+K_cA}$$

$$\frac{T'}{T_i'} = \frac{\frac{1}{1+K_cA}}{\frac{\tau s}{1+K_cA} + \frac{1+K_cA}{1+K_cA}} = \frac{\frac{1}{1+K_cA}}{\frac{\tau s}{1+K_cA}+1} = \frac{A_2}{\tau_1 s+1}$$

The overall response is first-order.
The overall time constant τ_1 is the same as for set-point changes. The response of the system to a unit-step change in inlet temp T_i', T' approaches $\frac{1}{(1+K_cA)}$. To demonstrate the benefit of control, the response of the tank temp. (Open-loop response) to a unit step change in inlet temp. if no control were present. i.e. $K_c = 0$. In this case, the major advantage of control is in reduction of offset.

$$T_i' = \frac{1}{s}$$

$$T' = \frac{1}{s} \frac{1/(1+K_cA)}{[\tau/(1+K_cA)]s+1}$$

Ultimate s.s value of T' is

$$T'(\infty) = \lim_{s\to 0} sT'(s) = \frac{1}{1+K_cA}$$

The offset becomes,

$$\text{offset} = T_R'(\infty) - T'(\infty) = 0 - \frac{1}{1+K_cA}$$

$$= -\frac{1}{1+K_cA}$$

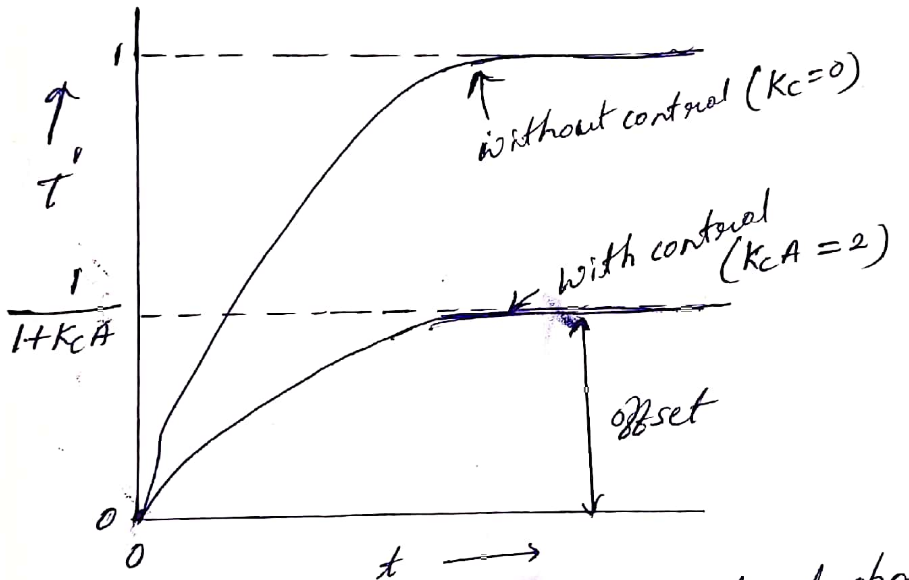


Fig - Unit-step response for load change (P control)

As for the case of a step change in set point, the offset is reduced as controller gain K_c is increased.

Proportional - Integral Control for Load Change. ($T_R'=0$)

For this type of controller, G_c is replaced by $K_c\left(1+\frac{1}{\tau_I s}\right)$

The overall transfer function for load change is,

$$\therefore \frac{T'}{T_i'} = \frac{AA^{-1}/(\tau s+1)}{1+[K_cA/(\tau s+1)]\left(1+\frac{1}{\tau_I s}\right)} \quad \text{--- ①}$$

Rearranging

$$\frac{T'}{T_i'} = \frac{\tau_I s}{(\tau s+1)(\tau_I s) + K_c A(\tau_I s+1)}$$

$$\frac{1/(\tau s+1)}{1+\left(\frac{K_c A}{\tau s+1}\right)\left(1+\frac{1}{\tau_I s}\right)}$$

$$\frac{1/(\tau s+1)}{1+\left(\frac{K_c A}{\tau s+1}\right)+\frac{K_c A}{(\tau s+1)(\tau_I s)}}$$

$$\frac{1/(\tau s+1)}{\frac{(\tau s+1)(\tau_I s)+K_c A(\tau_I s)+K_c A}{(\tau s+1)(\tau_I s)}}$$

$$\frac{\tau_I s}{(\tau s+1)(\tau_I s+1)+K_c A(\tau_I s+1)}$$

(or)

$$\frac{T'}{T_i'} = \frac{\tau_I s}{\tau \tau_I s^2 + (K_c A \tau_I + \tau_I)s + K_c A}$$

Since the denominator contain a quadratic expression, the transfer function may be written in the std. form of the transportation lag to give.

$$\frac{T'}{T_i'} = \frac{(\tau_I / K_c A)s}{(\tau \tau_I / K_c A)s^2 + \tau_I(1+1/K_c A)s+1}$$

(or)

$$\frac{T'}{T_i'} = \frac{A_1 s}{\tau_1^2 s^2 + 2\zeta\tau_1 s+1} \quad \text{--- (2)}$$

where,

$$A_1 = \frac{\tau_I}{K_c A}$$

$$\tau_1 = \sqrt{\frac{\tau \tau_I}{K_c A}}$$

$$\zeta = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau}}\,\frac{1+K_c A}{\sqrt{K_c A}}$$

For a unit-step change in load;

$$T_i' = \frac{1}{s}$$

Combining with eq (2).

$$T' = \frac{A_1 s}{s(\tau_1^2 s^2 + 2\zeta\tau_1 s+1)}$$

$$T' = \frac{A_1}{\tau_1^2 s^2 + 2\zeta\tau_1 s+1} \quad \text{--- (3)}$$

$$2\zeta\tau_1 = \tau_I\left(1+\frac{1}{K_c A}\right)$$

$$\zeta = \frac{1}{2}\frac{\tau_I}{\tau_1}\left(1+\frac{1}{K_c A}\right)$$

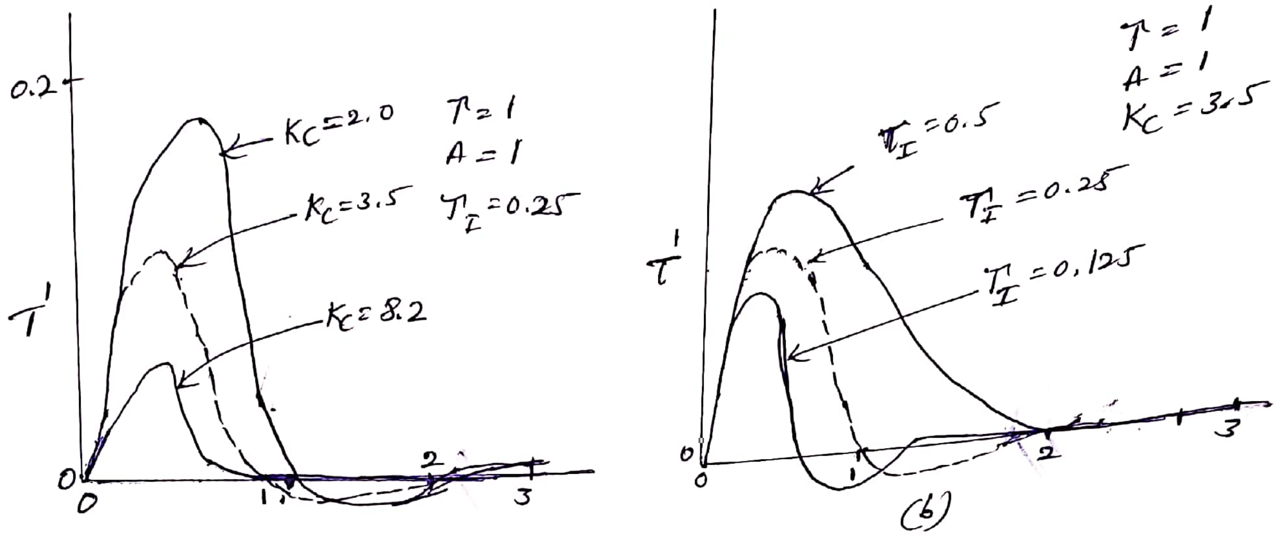
$$= \frac{1}{2}\frac{\tau_I\sqrt{K_c A}}{\sqrt{\tau \tau_I}}\left(\frac{K_c A+1}{K_c A}\right)$$

$$= \frac{1}{2}\sqrt{\frac{\tau_I}{\tau}}\,\frac{(1+K_c A)}{\sqrt{K_c A}}$$

This shows that response of the tank temp is equivalent to the response of a 2nd-order system to an impulse function of magnitude A_1.

The impulse response for this system for $\zeta<1$ as

$$T' = A_1\left(\frac{1}{\tau_1}\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta t/\tau_1}\sin\sqrt{1-\zeta^2}\,\frac{t}{\tau_1}\right) \quad \text{--- (4)}$$



(b)

(a)

Fig - Unit-step response for load change (PI control)

From fig. a., An increase in K_c for a fixed value of τ_I, improves the response by decreasing the maximum deviation and by making the response less oscillatory. The formula for ζ shows that ζ increases with K_c, which indicates that the response is less oscillatory.

Fig. b. For a fixed value of K_c, a decrease in τ_I decreases the maximum deviation and period. However, a decrease in τ_I cause the response to become more oscillatory, which means that ζ decreases. ~~This effect of τ_I on the oscillatory nature of the response is~~

$$\text{offset} = T_R'(\infty) - T'(\infty)$$
$$= 0 - 0 = 0$$

One of the most important advantages of PI control is the elimination of offset.

Proportional - Integral Control for Set - Point Change. $T_i' = 0$

The controller transfer function is $K_c\left(1+\frac{1}{\tau_I s}\right)$.

$$\frac{T'}{T_R'} = \frac{K_c A(1+1/\tau_I s)[1/(\tau s+1)]}{1+K_c A(1+1/\tau_I s)[1/(\tau s+1)]}$$

$$= \frac{\dfrac{K_c A(\tau_I s+1)}{(\tau_I s)(\tau s+1)}}{\dfrac{(\tau_I s)(\tau s+1)+K_c A(\tau_I s+1)}{(\tau_I s)(\tau s+1)}}$$

$$= \frac{K_c A(\tau_I s+1)}{\tau_I \tau s^2+(K_c A \tau_I+\tau_I)s+K_c A}$$

$\div \frac{Nr}{Dr}$ by $K_c A$

On simplification.

$$\frac{T'}{T_R'} = \frac{\tau_I s+1}{\tau_1^2 s^2+2\zeta\tau_1 s+1}$$

where, τ_1 and ζ are the same functions of the parameters. i.e

$$\tau_1 = \sqrt{\frac{\tau\tau_I}{K_c A}}, \quad \zeta = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau}}\,\frac{1+K_c A}{\sqrt{K_c A}}$$

Introducing a unit-step change $\left(T_R' = \frac{1}{s}\right)$

$$T' = \frac{1}{s}\cdot\frac{\tau_I s+1}{\tau_1^2 s^2+2\zeta\tau_1 s+1}$$

To obtain the response of T' in the time domain

$$T' = \frac{\tau_I}{\tau_1^2 s^2+2\zeta\tau_1 s+1} + \frac{1}{s}\,\frac{1}{\tau_1^2 s^2+2\zeta\tau_1 s+1}$$

The first term - equivalent to the response of a second-order system to an impulse function of magnitude τ_I.

Second term - is the unit-step response of a second-order system.

For $\zeta < 1$

$$T' = \frac{\tau_I}{\tau_1\sqrt{1-\zeta^2}}e^{-\zeta t/\tau_1}\sin\sqrt{1-\zeta^2}\,\frac{t}{\tau_1} + 1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\frac{t}{\tau_1}}\sin\left(\sqrt{1-\zeta^2}\,\frac{t}{\tau_1}+\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

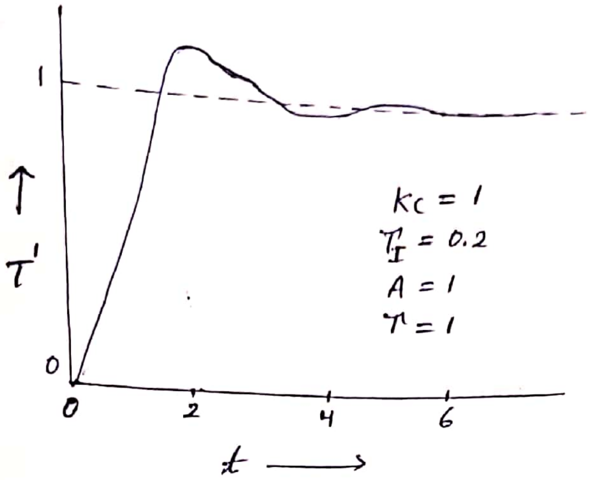


Fig - Unit step response for set point change (PI control).

$$\text{offset} = T_R'(\infty) - T'(\infty) = 1 - 1 = 0$$

The integral action in the controller has eliminated the offset.

Proportional Control of System with Measurement Lag

The lag in the measuring element was assumed to be negligible for which case the feedback transfer function was taken as 1.

Consider the stirred-tank heater with a first order measuring element having a transfer function $\left(\frac{1}{(\tau_m s+1)}\right)$.

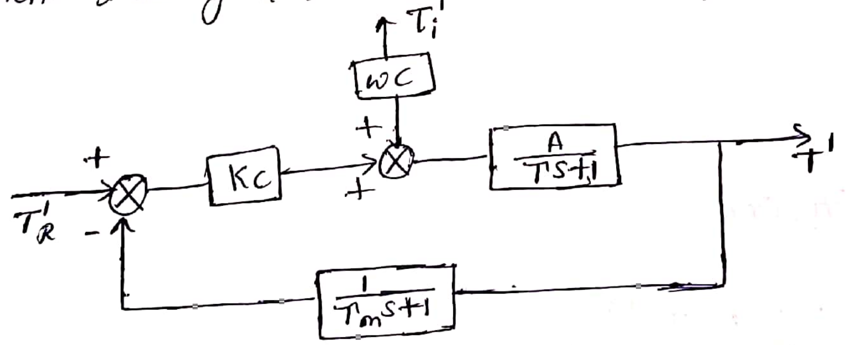


Fig - Control system with measurement lag.

The transfer function for set-point changes is written as,

$$\frac{T'}{T_R'} = \frac{A_1(\tau_m s+1)}{\tau_2^2 s^2 + 2\zeta_2\tau_2 s + 1}$$

where,

$$A_1 = \frac{K_c A}{1+K_c A}$$

$$\tau_2 = \sqrt{\frac{\tau\tau_m}{1+K_c A}}$$

$$\zeta_2 = \frac{\tau+\tau_m}{2\sqrt{\tau\tau_m}} \quad \frac{1}{\sqrt{1+K_c A}}$$

$$\frac{T'}{T_R'} = \frac{K_c A/(\tau s+1)}{1+\frac{K_c A}{(\tau s+1)(\tau_m s+1)}}$$

$$= \frac{K_c A/\cancel{(\tau s+1)}}{\frac{(\tau s+1)(\tau_m s+1)+K_c A}{\cancel{(\tau s+1)}(\tau_m s+1)}}$$

$$= \frac{K_c A(\tau_m s+1)}{(\tau s+1)(\tau_m s+1)+K_c A}$$

$$= \frac{K_c A(\tau_m s+1)}{\tau\tau_m s^2 + \tau s + \tau_m s + 1 + K_c A}$$

$\div\ \cancel{1} + K_c A$

PTB

$$\frac{\frac{K_c A}{(1+K_c A)}(\tau_m s+1)}{\frac{\tau\tau_m}{(1+K_c A)}s^2+\frac{\tau+\tau_m}{(1+K_c A)}\cdot s+\cancel{\frac{1+K_c A}{(1+K_c A)}}}$$

$$\frac{A_1(\tau_m s+1)}{\tau_2 s^2+2\zeta_2\tau_2 s+1}$$

$$2\zeta_2\tau_2=\frac{\tau+\tau_m}{(1+K_c A)}$$

$$\zeta_2=\frac{1}{2}\sqrt{\frac{1+K_c A}{\tau\tau_m}}\left(\frac{\tau+\tau_m}{1+K_c A}\right)$$

$$=\frac{\tau+\tau_m}{2\sqrt{\tau\tau_m}}\cdot\frac{1}{\sqrt{1+K_c A}}$$

Adding the first-order measuring lag to control system of stirred tank heater produces a second-order system even for proportional control. This means there will be an oscillatory response for an appropriate choice of the parameters τ, τ_m, K_c and A.

The response usually becomes more oscillatory or less stable, as K_c or τ_m increases.

For a fixed value of $\tau_m = 1$, fig shows that the offset is reduced as K_c increases. As K_c increases, the overshoot becomes excessive and the response becomes more oscillatory. In general, control system having proportional control will require a value of K_c that is based on a compromise between low offset and satisfactory transient response.

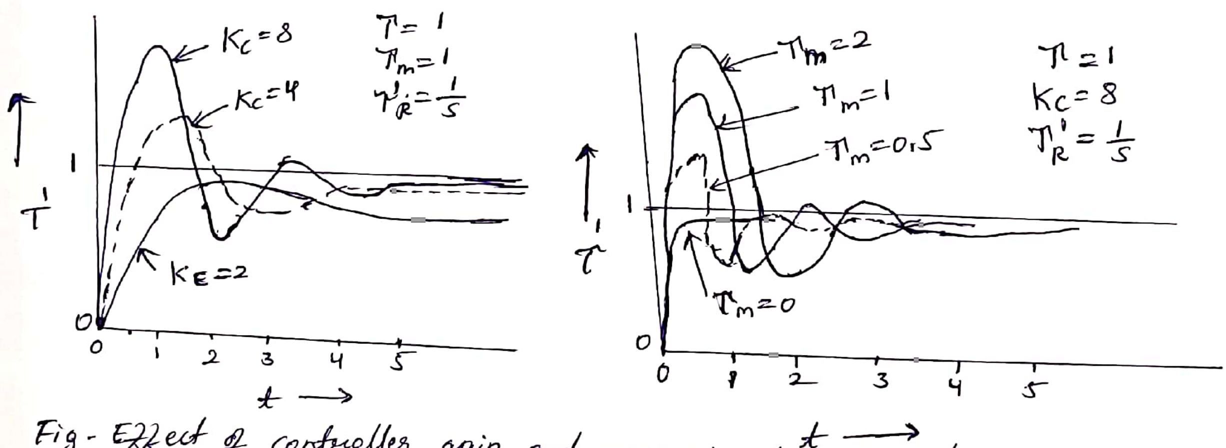
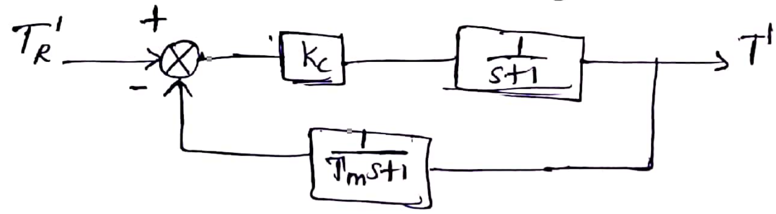


Fig - Effect of controller gain and measuring lag on system response for unit-step change in set point.

For a fixed value of controller gain ($K_c = 8$). fig shows that an increase in measurement lag produces a poorer transient response in that the overshoot becomes greater and the response more oscillatory as τ_m increases. This behavior illustrates a general rule that the measuring element in a control system should respond quickly if satisfactory response is to be achieved.



$A = 1$